1. Determine the factor group G/N for the given group G and normal subgroup N.
 a) $G = GL(n, K)$, $N = SL(n, K)$;
 b) $G = D_4$, $N = \langle f^2 \rangle$;
 c) $G = (\mathbb{R}, +)$, $N = \mathbb{Z}$;
 d) $G = Q^\times$, $N = \{\pm 1\}$;
 e) $G = \langle a \rangle \times \langle b \rangle$, ahol $o(a) = 4$ és $o(b) = 6$, $N = \langle a^2 b^3 \rangle$.

2. Let $N \triangleleft G$, $H \leq G$, $|G| = 24$, $|N| = 4$, and $|H| = 6$. What can be the order of the image of H under the action of the homomorphism $G \to G/N$? Give an example for each case.

3. Let $Q = \{\pm 1, \pm i, \pm j, \pm k\}$ be the quaternion group, where $ij = k$, $jk = i$, $ki = j$ $ji = -k$, $kj = -i$, $ik = -j$, $i^2 = j^2 = k^2 = -1$, and multiplication by -1 takes each element into its negative. Check that Q is a indeed a group and give the multiplication table for Q. Prove that every subgroup of Q is normal. Prove that Q cannot be obtained as a semidirect product of smaller groups.

4. For each $s \mid n$ prove that the dihedral group D_s is both a subgroup and a homomorphic image of the dihedral group D_n.

5. Determine the number of elements of order 4 in A_8.

6. Prove that A_n is generated by its k-cycles if k is odd, and $3 \leq k \leq n$.

7. Prove that S_4 has only four normal subgroups: 1, S_4, A_4, and the four element Klein group (containing the elements of the form (..)(..) and the identity).

8. Prove that A_4 has no subgroup of order 6.

9. Determine the conjugacy classes of A_5

HW1. Let N be a normal subgroup and H a subgroup in a group G of order 100, and suppose that $|N| = 20$ és $|H| > 20$. Prove that H has a subgroup of index 5.

HW2. Prove that the diagonal subgroup $T = \{(g, g) \mid g \in G\}$ of the direct product $G \times G$ is normal if and only if G is Abelian.

HW3. Find the smallest n such that S_n or A_n, respectively, contains a subgroup isomorphic to the 8-element dihedral group D_4.