1. Let $P \in Syl_p(G)$, and $H \leq G$ a p-group. Prove that $H \leq N_G(P)$ implies $H \leq P$.

2. Prove the second Sylow theorem (i. e. $|Syl_p(G)| \equiv 1 \,(\text{mod p})$), using the following hints. Consider the action of a Sylow p-subgroup P by conjugation on $Syl_p(G)$. Show that every orbit of P has p-power number of elements, and there is only one orbit containing exactly one element (see problem 1).

3. Prove that any p-subgroup H of a finite group G is contained in some Sylow p-subgroup of G. Use the second Sylow theorem and the sizes of orbits under the action of conjugation by elements of H.

4. a) What can be the number of Sylow 3-, 5-, and 7-subgroups of a group of order 105?
b) Prove that such a group has a normal Sylow subgroup.

5. Show that a group of order 72 cannot be simple.

6. Determine the number of Abelian groups of the following order up to isomorphism:
a) 32,
b) 360.

7. Determine the number of subgroups of order 12 in the Abelian group $C_4 \times C_2 \times C_9$. How many of these subgroups are cyclic?

8. Prove the following statements about the commutator subgroup.
a) $(G \times H)' = G' \times H'$;
b) if $H \leq G$ then $H' \leq G' \cap H$;
c) $\varphi(G') = (\varphi(G))'$ for any homomorphism $\varphi : G \to H$.

9. Determine the center and the commutator subgroup of the following groups:

a) S_n b) D_n c) a non-Abelian group of order p^3 d) $\left\{ \begin{bmatrix} a & b \\ 0 & c \end{bmatrix} \;\middle|\; a, b, c \in \mathbb{Z}_3,\; a, c \neq 0 \right\}$

10. Let H be a maximal subgroup of G. Prove that $H \geq Z(G)$ or $H \geq G'$.

HW1. Prove that a group of order 200 cannot be simple.

HW2. Determine, up to isomorphism, all Abelian groups of order 400 containing no element of order.

HW3. Determine the commutator subgroup of A_4.