1. Prove that G is solvable if the order of G is
a) p^2q^2, with p és q different primes;
b) pqr, with p, q, r different primes;
c*) p^3q, with p and q different primes.

2. Give a composition series for the dihedral group D_n and the group $GL(3,2)$.

3. Prove the following isomorphisms for the given finitely presented groups.
a) $\langle x, y, z \,|\, x^2 = y^2 = z^3 = 1,\ xy = yx,\ z^{-1}xz = y \rangle \cong A_4$;
b) $\langle x, y \,|\, x^2 = y^2 = 1,\ xyxy = yxyx \rangle \cong D_4$;
c) Every finite non-Abelian homomorphic image of $\langle x, y \,|\, x^2 = y^2 = 1 \rangle$ is isomorphic to one of the dihedral groups.

4. Give a presentation of S_4 with transpositions as generators. Give a presentation of A_5 with 3-cycles as generators.

5*. Prove that $\langle x, y, z \,|\, y^{-1}xy = x^2,\ z^{-1}yz = y^2,\ x^{-1}zx = z^2 \rangle = 1$.

6. Give a presentation for the following groups.
a) $C_2 \times C_2 \times C_4$ b) $C_3 \times C_8$ c) Q

7. Suppose that $x^2 = x$ for every element x of a ring R. Prove that R is commutative.

8. Let R be a ring with an identity and let $a \in R$ be a nilpotent element, i. e. $a^n = 0$ for some positive integer n. Prove that $1 + a$ is invertible.

9. Let R be a ring with $1 \in R$ and let $a, b \in R$ such that $1 + ab$ is invertible. Prove that $1 + ba$ is also invertible.

10. Prove that the invertible elements of a ring with an identity form a group with respect to multiplication. Do they (together with the 0 element) also form a (skew) field with respect to the ring operations?

11. Let R be a ring with an identity and with no zero divisors. Prove that the right inverse of an element is necessarily an inverse.

12. What can we say about the ring R if for every element $a \in R$ the set $\{0, a\}$ is an ideal of R?

HW1. Prove that the group G is solvable if $|G| = 8p$ with some odd prime p.

HW2. Prove that $\langle x, y \,|\, y^{-1}xy = x^2,\ x^{-1}yx = y^2 \rangle = 1$

HW3. Suppose that N is a normal subgroup of the group G, and $N \cap G' = 1$. Prove that $N \leq Z(G)$.