1. Consider the permutations $g = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 5 & 4 & 2 & 7 & 1 & 3 & 6 \end{pmatrix}$ and $h = (235)(14)$ in S_7. What is the disjoint cycle decomposition of g? Calculate gh and the conjugate $h^{-1}gh$, and then determine the order and parity (whether the permutations are even or odd) of the permutations g, h, gh and $h^{-1}gh$. (*7 points*)

2. Suppose g is an element of a non-commutative group G, furthermore, $|G| = 28$, $g^7 \neq 1$, and $g^8 \neq 1$. What can be the order of the element g? (*7 points*)

3. Let G be the group of invertible 2×2 upper triangular matrices over $\mathbb{Z}_5$, and $N = \left\{ \begin{bmatrix} a & b \\ 0 & a \end{bmatrix} \;\middle|\; a, b \in \mathbb{Z}_5,\ a \neq 0 \right\} \subseteq G$. Prove that N is a normal subgroup of G, and G/N is cyclic. (*7 points*)

4. What is the number of elements of order 12 in the alternating group A_{10}? (*7 points*)

5. a) Prove that if $\mathcal{C}_1$ and $\mathcal{C}_2$ are two conjugacy classes of a group G then the set $\mathcal{C}_1 \cdot \mathcal{C}_2 = \{\, gh \mid g \in \mathcal{C}_1,\ h \in \mathcal{C}_2 \,\}$ is the union of some conjugacy classes of G.

b) For $G = S_4$, the conjugacy class of 2-cycles, that is, $\mathcal{C}_1 = (12)^G$ and the conjugacy class of 3-cycles, that is, $\mathcal{C}_2 = (123)^G$, how many conjugacy classes of G does the set $\mathcal{C}_1 \cdot \mathcal{C}_2$ contain? (*7 points*)

6. Let $H, K \leq G$, $|G| = 80$, $|H| = 20$, $|K| = 8$. Prove that $\langle H \cup K \rangle \triangleleft G$. (*7 points*)