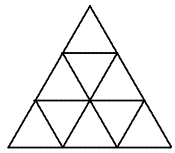
1. Consider the regular triangle below divided into 9 small congruent triangles. How many ways are there to colour three of the small triangles black, up to isometries of the big triangle?



(*7 points*)

2. What is the number of elements of order 4 in $D_4 \times Q$, where D_4 is the dihedral group of 8 elements and Q is the quaternion group? (*7 points*)

3. What is the number of abelian groups of order 200 up to isomorphism? List the canonical decompositions of those among them that have elements of order 4 but do not have elements of order 50? (*7 points*)

4. Let G be a group of order $3^3 \cdot 13$. What can be the number of Sylow 3- and 13-subgroups of G? Show that one of the Sylow-subgroups is normal.

5. Suppose that the finite group G has a normal subgroup N whose order is a p-power for some prime p. Show that every Sylow p-subgroup of G contains N. (*7 points*)

6. a) Let I be an ideal in the ring R, and $J = \{\, b \in R \mid ab = 0 \ \forall a \in I \,\}$. Prove that $J \triangleleft R$.

b) What is J if R is the group of 3×3 upper triangular matrices over $\mathbb{R}$, and I is the ideal of strictly upper triangular matrices, that is, where all the diagonal elements are zero. (*7 points*)