

## Reminder before the midterm test

### Concepts:

- subspace and spanned subspace, linear independence, generating set, basis, dimension of a vector space, coordinate vector
- linear map and transformation, their matrices in a given basis or pair of bases, similarity of matrices
- image and kernel of a linear map, rank of a linear map or matrix
- eigenvalue, eigenvector, eigenspace, spectrum
- characteristic polynomial, minimal polynomial
- Jordan block, Jordan matrix
- standard scalar product in  $\mathbb{R}^n$  and  $\mathbb{C}^n$ , orthogonal and orthonormal systems
- adjoint of a matrix, self-adjoint/symmetric, unitary/orthogonal and normal matrices
- definiteness of a real symmetric matrix
- definition of pseudoinverse by its properties
- singular value, reduced and full SVD

### Theorems:

- polynomial interpolation
- change of bases for matrices of linear maps and transformations
- Cayley–Hamilton theorem, connection between the minimal and the characteristic polynomial
- minimal polynomial and eigenvalues
- condition for diagonalizability by eigenvectors and by the minimal polynomial
- existence and uniqueness of the Jordan normal form
- connection between the Jordan normal form and the characteristic polynomial, the minimal polynomial and the dimension of the eigenspaces
- eigenvalues of unitary and self-adjoint transformations
- conditions for a matrix to be unitary
- equivalent characterizations of normal matrices (the spectral theorem)
- equivalent characterizations of self-adjoint and real symmetric matrices (corollaries of the spectral theorem)
- best approximating solution of an inconsistent system of equations by using the pseudoinverse
- applications of the SVD: calculating the pseudoinverse; approximation of a matrix by a low rank matrix

### Algorithms, computational methods:

- finding a basis of a spanned subspace, or of the image or kernel of a linear map, calculating the rank of a matrix
- Newton interpolation
- matrix of a transformation in a given basis, transition to another basis
- spectral decomposition ( $A = PDP^{-1}$ , where  $D$  is diagonal), diagonalization, calculating powers of a diagonalizable matrix

- calculating eigenvalues and eigenvectors
- operations of block matrices
- determining the Jordan normal form when the multiplicities of the eigenvalues is at most 6
- determine the invariants of the matrix from the Jordan normal form
- determining the definiteness of a symmetric matrix
- orthogonal projection of a vector on a vector in a real or complex Euclidean space
- finding the matrix of an orthogonal projection or reflection on a hyperplane in a real or complex Euclidean space
- calculating the reduced or full SVD of a real matrix
- calculating the pseudoinverse by SVD
- best approximating solution of an inconsistent system of equations
- low rank approximation of matrices by SVD