1. Is there a 3×3 matrix over $\mathbb{Q}$ with minimal polynomial

a) $x^2 - 2$;
b) $x^2 + x$?

2. Suppose that A is a matrix over $\mathbb{C}$ such that $A^m = I$ for some $m \geq 1$. Prove that A is diagonalizable.

3. Which of the following matrices are diagonalizable over $\mathbb{C}$? Determine the Jordan normal form of the matrices.

$$A = \begin{bmatrix} -3 & 1 & 2 \\ 1 & 1 & 0 \\ 2 & 0 & -1 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 0 & 3 \\ 0 & 1 & 0 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 3 & 1 \\ 0 & 0 & 0 & 4 \end{bmatrix} \qquad D = \begin{bmatrix} 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 2 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

4. What is the maximal number of non-similar complex matrices satisfying the following conditions? Give the Jordan normal form in each possible case.

a) $k(x) = -x^5(x+1)^2, \;\; m(x) = x^3(x+1)$;
b) $k(x) = (x-1)^4 x$, and the eigenspace for the eigenvalue 1 is 2-dimensional.

5. Find two non-similar 7×7 matrices which have the same minimal and characteristic polynomials, and their eigenspaces also have the same dimension.

6. Calculate the nth power of the following matrices, using the diagonal or Jordan normal form.

$$A = \begin{bmatrix} 5 & -6 \\ 3 & -4 \end{bmatrix} \qquad B = \begin{bmatrix} 4 & -4 \\ 1 & 0 \end{bmatrix}$$

7. Prove that every $n \times n$ complex matrix is similar to its transposed matrix. (Use the Jordan normal form.)

8. Is there a matrix $I \neq A \in \mathbb{Q}^{n \times n}$ such that a) $A^3 = I$; b) $A^5 = I$? And in $\mathbb{Q}^{2 \times 2}$?