**1.** Determine the standard matrix of the linear transformation $f(x, y, z) = (x + z, y, x + y + z)$, the basis of $\mathrm{Ker}\, f$ and $\mathrm{Im}\, f$, and the matrix of $f$ in the basis $\mathcal{B} = \{(1, 0, 0), (0, 1, 1), (0, 2, 1)\}$

**2.** Find the diagonal form of the matrix $A = \begin{bmatrix} -1 & 1 \\ -3 & 3 \end{bmatrix}$, together with the transition matrix. Calculate $A^n$ for any natural number $n$.

**3.** What can be the Jordan normal form of a matrix $A$ whose characteristic polynomial is $k_A(x) = (x + 1)^4 x^2$ and whose minimal polynomial is $m_A(x) = (x + 1)^2 x^2$. Give the dimension of the eigenspaces is each case.

**4.** Which of the following matrices are self-adjoint, normal or unitary? Determine the definiteness of those which are self-adjoint.

$$A = \begin{bmatrix} i & 1 \\ -1 & 1+i \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & -1 \\ 1 & -1 & 1 \end{bmatrix} \qquad C = \begin{bmatrix} 1/\sqrt{5} & 0 & 2/\sqrt{5} \\ 0 & -1 & 0 \\ -2/\sqrt{5} & 0 & 1/\sqrt{5} \end{bmatrix}$$

**5.** Determine the reduced SVD of $A = \begin{bmatrix} 1 & 1 \\ -1 & 1 \\ 1 & 1 \end{bmatrix}$, and the best approximating matrix of rank 1.

**6.** State the following definitions and theorems
- a) rank of a matrix;
- b) the theorem of polynomial interpolation;
- c) standard scalar product in $\mathbb{C}$;
- d) theorem about the connection between diagonalizability and the minimal polynomial of a matrix;
- e) singular values.