*Show all the details of your work in your solution. You may use a calculator or a calculator application, and you are free to look at the lecture notes or the exercises and their solutions. But do not communicate with each other or other people during the test. You can reach me if necessary through Teams chat or email:* `lukacs@math.bme.hu`.

1. Determine the link matrix A of the following graph defined in the PageRank algorithm. Find a ranking as a positive eigenvector of matrix A for eigenvalue 1. (4 marks)
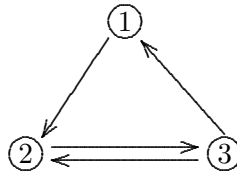
2. Find the eigenvalues and eigenvectors of the matrix below. Using these, find the matrices P and D such that $A = PDP^{-1}$, where D is a diagonal matrix. (You do not have to calculate P^{-1}.) (6 marks)

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 1 & 0 & 2 \\ -1 & 0 & -2 \end{bmatrix}$$

3. Determine whether the following matrices are self-adjoint, unitary or normal (this means 6 answers!). Justify each of your answers. (5 marks)

$$A = \begin{bmatrix} 1 & 1-i \\ 1+i & 2 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$$

4. Orthogonalize the column vectors of the following matrix A by Gram–Schmidt's method. Use this to give the (reduced) QR decomposition of A. (5 marks)

$$A = \begin{bmatrix} 1 & 3 \\ -1 & 1 \\ 1 & -1 \\ 1 & 3 \end{bmatrix}$$