1. Determine the polar form of $1-3i$. What is $(1/(1-3i))^2+(1/(1+3i))^2$?

2. Let $z, w \in \mathbb{C}$ such that $|z|=5$, $|w|=3$. What is $|zw|$, $|z/w|$, $|z+w|$, $|z-w|$? If some cannot be determined exactly then give lower and upper bounds for them.

3. True or false?

(i) Every complex number has a square root.

(ii) Every complex number has a 100-th root.

(iii) If a complex number is not real then its square root is not real.

(iv) If a complex number is purely imaginary then its square root is purely imaginary.

(v) The square root(s) of a negative real number are purely imaginary.

4. Find the eigenvalues and eigenvectors of the following rank 1 matrices:

$$A=\begin{bmatrix}0\\1\\2\end{bmatrix}\begin{bmatrix}3 & -2 & 0\end{bmatrix}=\begin{bmatrix}0 & 0 & 0\\3 & -2 & 0\\6 & -4 & 0\end{bmatrix}; \qquad B=\begin{bmatrix}1\\\pi\\-\pi\end{bmatrix}\begin{bmatrix}\sqrt{2} & -1 & -1\end{bmatrix}; \qquad C=\frac{\mathbf{x}\mathbf{y}^T}{\mathbf{x}^T\mathbf{y}}.$$

5. Simon says: "every triangular matrix has its eigenvalues on the diagonal." Is he right?

6. **HW** Find four independent eigenvectors of D and thus diagonalise it!

$$D=\begin{bmatrix}0 & 1 & 1 & 1\\1 & 0 & 1 & 1\\1 & 1 & 0 & 1\\1 & 1 & 1 & 0\end{bmatrix}.$$

7. Find the eigenvalues of the following permutation matrix:

$$P=\begin{bmatrix}0 & 0 & 0 & 1\\1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 1 & 0\end{bmatrix}.$$

8. Suppose $AX=X\Lambda$ with X invertible. Which is true?

(i) If X is triangular then A is triangular.

(ii) If A is triangular then X is triangular.

(iii) The columns of $(X^{-1})^T$ are eigenvectors of A^T.

9. Let $g_0=0$ and $g_1=1$. For $n>1$ define $g_n=\frac{g_{n-1}+g_{n-2}}{2}$. (This sequence of consecutive means is sometimes called "Gibonacci" series.) Using matrices determine a formula for g_n and show that $g_n \to 2/3$.

10. Diagonalise A to $X^{-1}AX=\Lambda$ and determine the limits $\Lambda^n \to \Lambda^\infty$ and $A^n \to A^\infty$.

$$A=\begin{bmatrix}0 & 0.2 & 0.4\\0.3 & 0.4 & 0.5\\0.7 & 0.4 & 0.1\end{bmatrix}$$