1. Determine the cubic roots of $-2+2i$.

2. Find all complex numbers z such that $z^2+(2i-2)z-2-2i=0$.

3. True or false?

(i) If A is similar to I then $A=I$.

(ii) If A is similar to $2A$ then $A=0$.

(iii) There are infinitely many regular 2×2 matrices A similar to A^{-1}.

4. Connect similar matrices (only).

$$A=\begin{bmatrix} 0 & 0 & 0 \\ 3 & -2 & 0 \\ 6 & -4 & 0 \end{bmatrix};\ B=\begin{bmatrix} 0 & 0 & 0 \\ 3 & 2 & 0 \\ 6 & 4 & 0 \end{bmatrix};\ C=\begin{bmatrix} 0 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 0 \end{bmatrix};\ D=\begin{bmatrix} -2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix};\ E=\begin{bmatrix} -\lambda & 0 & 0 \\ 3 & -2-\lambda & 0 \\ 6 & -4 & -\lambda \end{bmatrix}.$$

5. Simon says: "Eigenvectors for $\lambda=0$ span the nullspace and eigenvectors for $\lambda\neq 0$ span the column space." in what is he right, if at all?

6. HW We found that a 2×2 real matrix A must have $\mathrm{tr}(A)<0$ and $\det(A)>0$ to be stable (that is for the solution of $\mathbf{u}'(t)=A\mathbf{u}(t)$ we have $\mathbf{u}(t)\to 0$, if $t\to\infty$). What does it say about

$$A=\begin{bmatrix} a & b \\ c & 0 \end{bmatrix}?$$

Check the eigenvalues and their real parts.

7. Find the eigenvalues and the eigenvectors of A to solve the following system of linear differential equations:

$$\frac{d\mathbf{u}}{dt}=\begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}\mathbf{u},\qquad \text{that is, } \mathbf{u}(t)=\begin{bmatrix} p(t) \\ q(t) \end{bmatrix},\qquad \begin{aligned} dp/dt &= 2p+q \\ dq/dt &= q \end{aligned}$$

if $\mathbf{u}(0)=\begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

8. Suppose A, B are matrices that are diagonalised by the same X. Show that $AB=BA$.

9. Let $f_n=f_{n-1}-f_{n-2}+f_{n-3}$ be a linear recursion with initial values: $f_1=1, f_2=2, f_3=3$. Determine f_{100} using the matrix method.

10. Solve the differential equation $y''(t)=3y'(t)-2y(t)$ with initial values $y'(0)=1$, $y(0)=0$ using the matrix method.

11. Leftland and Rightland are neighbouring countries with open borders. Their populations $l(t)$ and $r(t)$ satisfy (t is measured in years)

$$\begin{aligned} dl/dt &= r-l \\ dr/dt &= l-r \end{aligned}.$$

Comment on what phenomenon these describe. Suppose that all 10^8 people celebrate the Third Millennium at Leftland (with Rightland left empty) and then they are subjected to these laws. What will be the situation after 10 years? Are these functions convergent at $t\to\infty$?

12. Let $A=\begin{bmatrix} 2 & 2 \\ 0 & 0 \end{bmatrix}$. Diagonalise A and determine e^{At}.