1. Determine the eigenvalues and an orthonormal set of eigenvectors of S and write it $S = Q\Lambda Q^T$.

$$S = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}.$$

2. Find the/a Schur decomposition of

$$B = \begin{bmatrix} 2 & -1 \\ 1 & 4 \end{bmatrix}.$$

3. True or false?

(i) If A has n orthogonal eigenvectors then it has n orthonormal eigenvectors.

(ii) If A is real 2×2 and its determinant is negative then A has a positive and a negative pivot.

(iii) If A is real 2×2, its determinant is negative and has two orthonormal eigenvectors then A has a positive and a negative pivot.

(iv) If A is symmetric and $A^{100} = 0$ then $A = 0$.

4. Find an a so that the following matrix has a negative eigenvalue.

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & a \\ 1 & a & 2 \end{bmatrix}.$$

How many negative eigenvalues can A have? What are the signs of the pivots? What are the signs of the LPM's?

5. Simon says: "If A is symmetric then $N(A)$ and $C(A)$ are orthogonal subspaces." Is he right? Why? He also says: "No positive definite matrix has a 0 on the main diagonal!" Is he right now? Why?

6. **HW** Find all orthogonal matrices that diagonlise

$$A = \begin{bmatrix} 31 & -8 \\ -8 & 19 \end{bmatrix}?$$

7. Show that if a and b are chosen so that A and B are Positive Definite then C is also Positive Definite.

$$A = \begin{bmatrix} 1 & 3 \\ 3 & a \end{bmatrix}; \qquad B = \begin{bmatrix} 4 & b \\ b & 6 \end{bmatrix}; \qquad C = \begin{bmatrix} a & b \\ b & a \end{bmatrix}.$$

Determine the decomposition $C = LDL^T$ if $a = 25$ and $b = 20$. Use it to find M such that $C = M^T M$ (Cholesky factorisation).

8. Let

$$S = \begin{bmatrix} \cos\vartheta & -\sin\vartheta \\ \sin\vartheta & \cos\vartheta \end{bmatrix} \begin{bmatrix} 4 & 0 \\ 0 & 6 \end{bmatrix} \begin{bmatrix} \cos\vartheta & \sin\vartheta \\ -\sin\vartheta & \cos\vartheta \end{bmatrix}.$$

What is the determinant of S, what are the eigenvalues and eigenvectors of S? Is S Positive Definite?

9. Find the singular values and the SVD of $A = \begin{bmatrix} -1 & 2 \\ 2 & -4 \end{bmatrix}$. What are the eigenvalues of A?

10. What are the singular values and the SVD of the matrix $B = \begin{bmatrix} 1 & 2 & -3 \end{bmatrix}$?