1. True or false?

(i) Vectors are dependent if their dot product is 0.

(ii) If the linear combinations of $\mathbf{v}$ and $\mathbf{w}$ form a plane then $\mathbf{v}$ and $\mathbf{w}$ are independent.

(iii) $2\mathbf{v}$ is always longer than $\mathbf{v}$.

(iv) $\mathbf{a} - \mathbf{b}$ is the same as $\mathbf{a} + (-\mathbf{b})$.

(v) $\mathbf{v}$ and $\mathbf{w}$ are on a common line through the origin (called collinear) if and only if $\frac{\mathbf{v}}{||\mathbf{v}||} = \frac{\mathbf{w}}{||\mathbf{w}||}$.

(vi) Any vector is a matrix.

2. Let

$$\mathbf{a} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \mathbf{c} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \mathbf{d} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}.$$

Describe algebraically and geometrically the linear combinations of $\mathbf{b}$ and $\mathbf{c}$. Describe also the linear combinations of $\mathbf{a}$ and $\mathbf{b}$. Which of $\mathbf{c}$, $\mathbf{d}$ can be expressed using $\mathbf{a}$ and $\mathbf{b}$?

Determine the subsets of $\{\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}\}$ that are independent.

3. Let a clock have unit radius and let $\mathbf{a}_1, \dots \mathbf{a}_{12}$ denote the twelve vectors pointing from the centre of the clock (the origin) to the twelve round hours. Determine the entries of $\mathbf{a}_1$, $\mathbf{a}_4$ and $\mathbf{a}_9$. What is the sum $\mathbf{a}_1 + \mathbf{a}_2 + \cdots + \mathbf{a}_{12}$? What is the sum $\mathbf{a}_1 + \mathbf{a}_5 + \mathbf{a}_9$?

4. Find two vectors $\mathbf{a}, \mathbf{b}$ that are perpendicular to $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ and to each other! **HW** with $\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$.

5. Show these properties of the dot product:

(i) $\mathbf{v} \cdot \mathbf{w} = \mathbf{w} \cdot \mathbf{v}$;

(ii) $\mathbf{v} \cdot (\mathbf{w} + \mathbf{u}) = \mathbf{v} \cdot \mathbf{w} + \mathbf{v} \cdot \mathbf{u}$;

(iii) $(\mathbf{v} + \mathbf{w}) \cdot \mathbf{u} = \mathbf{v} \cdot \mathbf{u} + \mathbf{w} \cdot \mathbf{u}$.

6. Let $\mathbf{v} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$. Compute the real number(s) c for which $\mathbf{w} - c\mathbf{v}$ is perpendicular to $\mathbf{v}$. Can you do it for arbitrary $\mathbf{v}$ and $\mathbf{w}$?

7. Using Pythagoras' Theorem, prove the triangle inequality $||\mathbf{v} + \mathbf{w}|| \leq ||\mathbf{v}|| + ||\mathbf{w}||$.

8. How many vectors can you find on the plane/space such that every pair has negative dot product?

9. To every space vector $\mathbf{v}$ there corresponds a plane $P_{\mathbf{v}}$ consisting of the vectors perpendicular to $\mathbf{v}$: $P_{\mathbf{v}} = \{\mathbf{w} | \mathbf{w} \cdot \mathbf{v} = 0\}$. Describe the possible configurations of the planes corresponding to three independent vectors. Similarly, describe the possible configurations of the planes corresponding to three dependent vectors.

10. To every plane P (not necessarily containing the origin!) in the space there corresponds a (nonunique) vector $\mathbf{v}_P \neq \mathbf{0}$ perpendicular to P: $\mathbf{v}_P \cdot (\mathbf{w}_1 - \mathbf{w}_2) = 0$ for all $\mathbf{w}_1, \mathbf{w}_2 \in P$. Describe the configurations of three planes when the corresponding three vectors are independent and when they are dependent.