1. Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$. Write down the three elimination matrices E_{12}, E_{13}, E_{23} that turn A into upper triangular form. Which of these three matrices commute with each other? Compute the matrix product $E_{23}E_{13}E_{12}A$.

2. True or false?

(i) If every entry of A and B is positive then every entry of AB is also positive. (We assume AB exists.)

(ii) If elimination takes A to U then $A\mathbf{x} = \mathbf{0}$ implies $U\mathbf{x} = \mathbf{0}$.

(iii) If $A\mathbf{x} = \mathbf{0}$ has a nontrivial solution then A has no 0 entries.

(iv) If AB and BA are both defined then both of AB and BA are square.

3. Determine the "swap matrix" S_{ij} such that $S_{ij}A$ is the same as A, but its rows i and j are swapped. What is AS_{ij} (if A is $n \times n$)?

4. Multiply the following matrices A and B. First using row-times-column dot products and then using column-times-row matrices that you add up.

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & -1 \\ 1 & -1 \\ 0 & -2 \end{bmatrix}$$

5. Let E_{21} denote the elimination matrix that subtracts the double of row 1 from row 2 and let S_{23} denote the swap matrix of rows 2 and 3. What is $S_{23}E_{21}$, which does the two steps at once?

6. Write the following problem in a system of linear equations $A\mathbf{x} = \mathbf{b}$: Old Smith has a son, who is two years younger then half of his age. He also has a granddaughter, who is 4 years older than the third of her father's age. The sum of all of their ages is 118. How old are they?

7. Multiply the following matrices C and D as CD and as DC. Also compute C^{10} and D^{200}.

$$C = \begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & 0 & 1 \end{bmatrix} \qquad D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & c & 1 \end{bmatrix}$$

8. Show that for $n \times n$ matrices A, B in general we do not have $(A + B)^2 = A^2 + 2AB + B^2$. What should we write instead of the middle term?

9. **HW**(3×3 matrices) What is the matrix B such that for every matrix A

(i) $BA = 3A$;

(ii) every row of BA is equal to the first row of A?

10. For which numbers a, b, c, d does the following hold?

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 11 \\ 11 \end{bmatrix} = \begin{bmatrix} 11 \\ 11 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$