1. Compute the inverse of $\begin{bmatrix} 2 & 2 \\ 3 & 4 \end{bmatrix}$ using Gauss-Jordan's method.

2. True or false?

(i) If $A^T = A^{-1}$ then A is symmetric.

(ii) If $A^T A = AA^T$ then A is square.

(iii) If $A^T = -A$ then A is invertible.

(iv) Every elimination matrix is diagonally dominant, so invertible.

(v) Suppose in the 3×3 matrix A the sum of the first two columns is the third column. Then $A\mathbf{x} = \mathbf{0}$ surely has a nonzero solution.

3. Determine the inverse of the following two matrices $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$, $\quad B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 1 & 3 & 3 & 1 \end{bmatrix}$.

4. Suppose A, B, C are $n \times n$ and $ABC = D$ is invertible. Show that A, B, C are all invertible and express B^{-1} using D^{-1} and A, C.

5. What is a, b so that the following holds?

$$\begin{bmatrix} 4 & -1 & -1 & -1 \\ -1 & 4 & -1 & -1 \\ -1 & -1 & 4 & -1 \\ -1 & -1 & -1 & 4 \end{bmatrix}^{-1} = \begin{bmatrix} a & b & b & b \\ b & a & b & b \\ b & b & a & b \\ b & b & b & a \end{bmatrix}$$

Can you generalise?

6. Compute the LU decomposition of

$$C = \begin{bmatrix} 3 & 1 & 0 \\ 1 & 3 & 1 \\ 0 & 1 & 3 \end{bmatrix}.$$

Make it into an LDU decomposition.

7. Separate the following system $A\mathbf{x} = \mathbf{b}$ into $L\mathbf{y} = \mathbf{b}$ and $U\mathbf{x} = \mathbf{y}$.

$$\begin{aligned} x+ \quad y+ \quad z &= 1 \\ x+ \; 2y+ \; 3z &= 3 \\ x+ \; 3y+ \; 5z &= 10. \end{aligned}$$

8. HW Let $A = \begin{bmatrix} 1 & c & 0 \\ 2 & 4 & 1 \\ 3 & 5 & 1 \end{bmatrix}$. For which values of c does elimination break down and how? Write $A = LU$ in the other cases.

9. We mentioned that if A, B are lower triangular then their product AB is also lower triangular. Prove it! Also prove that if A is lower triangular with nonzero elements on the diagonal then the inverse A^{-1} is also lower triangular.

Hint for both: Consider the i-th column of AB being governed by the i-th column of B. Where are the 0's?