**1.** Let $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix}$. Is there a subspace of the vector space of $2 \times 2$ matrices that contains exactly one of $A$ and $B$?
Is there a subspace that contains exactly two of $A$, $B$ and $I$?
Is there a subsapce that contains no nonzero diagonal matrices?

**2.** True or false for an $m \times n$ matrix $A$ of rank $r$ describing a system $A\mathbf{x} = \mathbf{b}$.?

(i) If $n = m = r$ then $A$ is invertible.

(ii) If $n < m$ then the system has no solution.

(iii) If $n > m$ then there are free columns.

(iv) If the system has a unique solution then $n = r$.

(v) If the system has no solution then $\mathbf{b} \neq \mathbf{0}$ and $r > 0$.

**3.** Let $V = \mathbb{R}^2$ but scaling is defined by $\lambda \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} \lambda v_1 \\ 0 \end{bmatrix}$. Is this a vector space?

**4.** Let $V = \mathbb{R}^2$ but addition is defined by $\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} + \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} v_1 + w_2 \\ v_2 + w_1 \end{bmatrix}$. Is this a vector space?

**5.** Describe the smallest subspace of the $2 \times 3$ matrices that contain (a) $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$; (b) $A$ and $B = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \end{bmatrix}$; (c) $B$ and $C = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix}$.

**6.** Which of the following give the correct definition of the rank of $A$ (with $R$ being a reduced row echelon form)?

(i) The number of nonzero rows of $R$.

(ii) The number of columns minus the number of zero rows.

(iii) The number of columns minus the number of free columns.

(iv) The number of 1's in the matrix $R$.

**7.** Write the special solutions of $R\mathbf{x} = \mathbf{0}$ and of $R^T\mathbf{y} = \mathbf{0}$ for the following matrices. Write down the nullspace matrices.

$$R_1 = \begin{bmatrix} 1 & 0 & 2 & 4 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad R_2 = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

**8. HW** Find the reduced row echelon form and the rank of $A = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 2 & 4 \\ 1 & c & 2 \end{bmatrix}$. Which are the pivot columns? Give the special solutions. (The answer will depend on $c$.)

**9.** Prove that every rank-$r$ matrix can be written as a sum of $r$ rank-1 matrices!