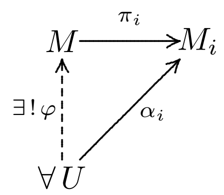
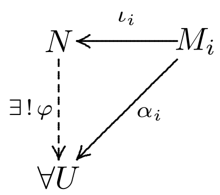
1. Prove that the following hold in every category:
 a) if $\alpha\beta$ is an epimorphism then β is an epimorphism;
 b) if $\alpha\beta$ is a monomorphis then α is a monomorphism.

2. Prove that the natural embedding $\mathbb{Z} \rightarrowtail \mathbb{Q}$ is an epimorphism in the category of rings with identity, though it is not surjective.

3. Prove that the product in a category is unique up to isomorphism and that in Mod-R, the product is the direct product.

M is a product: 

N is a coproduct: 

4. What are the epimorphisms and monomorphisms, injective and projective elements in the category of sets? Show that in this category the product and coproduct of two objects are usually not isomorphic.

5. What is the coproduct of $\mathbb{Z}$ with itself, in the category of Abelian groups and in the category of groups? What is the coproduct of $\mathbb{Z}_2$ with itself in the category of groups?

6. Prove that the direct product of injective modules is injective and that any direct summand of an injective module is also injective. Which of these statements can be generalized to any category?

7. Show that every vector space is projective and injective

8. Suppose that G is a divisible abelian group.
 a) Prove that for any element $g \in G$, if $o(g) = \infty$, then g is included in a direct summand of G isomorphic to $\mathbb{Q}$, if $o(g) = p^n$ for some p prime and $n \geq 1$, then g is included in a direct summand of G isomorphic to $\mathbb{Z}_{p^\infty}$.
 b) Prove that G is the direct some of some copies of $\mathbb{Q}$ $\mathbb{Z}_{p^\infty}$ (p any prime).

HW1. Prove that among the $\mathbb{Z}_4$-modules, $\mathbb{Z}_4$ is injective but $\mathbb{Z}_2$ is not.

HW2. Prove that in the category of sets the product is the cartesian product, and the coproduct is the disjoint union of sets.