1. Find the graph algebra which is isomorphic to the algebra of 2×2 upper triangular matrices over K.

2. Show that for $A = K^{n\times n}$, the regular module A_A is the direct sum of n isomorphic simple modules.

3. Let $M \in \text{mod-}A$ be indecomposable. Use the Fitting Lemma to prove that $R = \text{End}(M)$ is a local ring, i.e. $R/J(R)$ is a division ring.

4. Consider the map $M \mapsto D(M) = \text{Hom}_K(M, K)$ from the category mod-A to A-mod (or from A-mod to mod-A). Let D act on the morphisms so that for $\alpha : M \to N$, we have $D(\alpha) : D(N) \to D(M)$, where $\varphi D(\alpha) := \alpha\varphi$. Prove that this D defines a vector space isomorphism from $\text{Hom}(M, N)$ to $\text{Hom}(D(N), D(M))$ for any modules M, N, furthermore, $D(\alpha\beta) = D(\beta)D(\alpha)$ and $D(1_M) = 1_{D(M)}$ (so D is a contravariant functor from mod-A to A-mod).

5. Prove that in the setting of problem 4, $D^2(M) \cong M$ for every module, and D maps projective modules to injective modules and injective modules to projective modules.

6. Suppose that A is a graph algebra with arrows $\alpha_1, \ldots, \alpha_k$, and that the module $M \in \text{mod-}A$ has a basis $\mathcal{B} = \{ b_1, \ldots, b_\ell \}$ such that the arrows map the set $\mathcal{B} \cup \{ 0 \}$ into itself. Let us denote by $b_i\alpha^{-1}$ the set $\{ x \in \mathcal{B} \mid x\alpha = b_i \}$. Prove that $\mathcal{B}' = \{ b'_1, \ldots, b'_k \}$ is a basis of $D(M)$ if $b'_i : b_j \mapsto \delta_{ij}$, and for this basis $\alpha b'_i = (\sum b_i\alpha^{-1})'$.

7. Let $A_A = \begin{smallmatrix}1\\2\\2\end{smallmatrix} \oplus \begin{smallmatrix}2\\2\\2\end{smallmatrix}$. Determine the Loewy diagram of ${}_AA$, and by using the K-dual, determine all indecomposable injective right modules of A. Find the irreducible morphisms in ind-A going to projective or from injective modules.

8. Find the Loewy diagram of the K-dual of the modules $\begin{smallmatrix}&1\\1&2\\&2\end{smallmatrix}$ and $\begin{smallmatrix}1&2\\2&2\\&2\end{smallmatrix}$ over the algebra of problem 7.

Hf1. Prove that the functor D on finite dimensional A-modules maps injective morphisms to surjective morphisms, surjective morphisms to injective morphisms, and $D(M)$ is indecomposable if and only if M is indecomposable. (Remember problem 2/2.)

Hf2. Determine the Loewy diagrams of the indecomposable injective modules of the algebra $A_A = \begin{smallmatrix}1\\2\\3\end{smallmatrix} \oplus \begin{smallmatrix}&2&\\1&&3\end{smallmatrix} \oplus \begin{smallmatrix}3\\1\end{smallmatrix}$.