

Concepts

- short exact sequence, split short exact sequence
- Auslander–Reiten sequence
- Auslander–Reiten translate
- group representation as a module or as a group homomorphism
- equivalence of representations
- irreducible representation
- linear representation
- character, class function,
- degree of a representation/character
- kernel and center of a character
- trivial and regular character
- scalar product of class functions
- character table
- permutation character
- induced class function/character
- algebraic integer

Theorems

- the existence of the ARS
- connection between the ARS and the irreducible morphisms
- Maschke’s Theorem
- Schur’s Lemma
- $\dim_K D < \infty$, D is a division algebra $\Rightarrow D = K$
- $\text{End } S = K$ if $S \in \text{mod-}A$ is simple and K is algebraically closed
- $|A| = \sum n_i^2$ if A is semisimple and K is algebraically closed
- number of linear representations over \mathbb{C} .
- number of irreducible representations over \mathbb{C}
- basic properties of characters
- decomposition of the regular character
- the central orthogonal idempotents of $\mathbb{C}G$
- 1st and 2nd orthogonality relations
- $\text{Irr}(G)$ is an orthonormal a basis of $\text{Cl}(G)$
- equivalence of representations in terms of their characters
- describing characters and irreducible characters by the scalar product
- calculation of the induced character
- a class function induces a class function
- Frobenius reciprocity
- a character induces a character
- properties of the center of a character
- $\chi(g)|\mathcal{K}(g)|/\chi(1)$ is an algebraic integer if χ is irreducible
- degrees of irreducible characters divide $|G|$
- Burnside’s theorem about the $(\chi(1), |\mathcal{K}|) = 1$ case
- corollary of Burnside’s theorem for simple groups
- Burnside’s $p^\alpha q^\beta$ -theorem