NAME: ______________________________

Repr. of Rings and Groups **Test 1** *April 2, 2013*

I. rész. *In this part every answer is worth 3 marks. Write your answer in the box after the question. Explain only when it is required.*

1. *Show an abelian group whose radical is isomorphic to* $\mathbb{Z}_6$.

2. *Define the semisimplicity of a module. Give two statements which are equivalent to semisimplicity.*

3. *Show a* $\mathbb{Z}_4$*-module which is not injective and not local. Explain briefly why it is not injective.*

4. *Let* $A_A = \begin{smallmatrix} 1 \\ 2 \\ 1 \end{smallmatrix} \oplus \begin{smallmatrix} & 2 & \\ 1 & & 3 \\ & & 1 \end{smallmatrix} \oplus \begin{smallmatrix} 3 \\ 1 \end{smallmatrix}$. *Pick an irreduble morphism from the following nonzero morphisms. Briefly justify your answer.*

   a) $1 \to \begin{smallmatrix} 1 \\ 2 \\ 1 \end{smallmatrix}$ *b)* $\begin{smallmatrix} 3 \\ 1 \end{smallmatrix} \to \begin{smallmatrix} & 2 & \\ 1 & & 3 \\ & & 1 \end{smallmatrix}$ *c)* $\begin{smallmatrix} 1 \\ 2 \end{smallmatrix} \to \begin{smallmatrix} 3 \\ 1 \end{smallmatrix}$ *d)* $\begin{smallmatrix} 2 & & 3 \\ & 1 & \end{smallmatrix} \to 3$

5. *State the theorem about the connection of a direct decompositon of a ring* R *(as a module and as a ring) and the idempotent elements of* R.

6. *State the Fitting Lemma. Give an example for the necessity of one of the conditions of the lemma.*

7. *Define the Auslander–Reiten sequence.*

8. $A_A = \begin{smallmatrix} & 1 & \\ 2 & & 3 \end{smallmatrix} \oplus \begin{smallmatrix} 2 \\ 3 \end{smallmatrix} \oplus \begin{smallmatrix} 3 \\ 1 \\ 2 \end{smallmatrix}$ *is the direct sum of indecomposable projective modules and* $D({}_AA) = \begin{smallmatrix} 3 \\ 1 \end{smallmatrix} \oplus \begin{smallmatrix} 3 \\ 1 \\ 2 \end{smallmatrix} \oplus \begin{smallmatrix} 1 & & 2 \\ & 3 & \end{smallmatrix}$ *is the direct sum of the indecomposable injective modules of the algebra A. Determine the Auslander–Reiten translate of* $\begin{smallmatrix} 3 \\ 1 \end{smallmatrix}$.

II. rész.

9. *Give a condition equivalent to the injectivity of a module, and prove the equivalence.* (10 *marks*)

10. *State and prove the Harada–Sai Lemma.* (16 *marks*)