

NÉV: _____

Part I In this part every answer is worth 3 marks. Write your answer in the box after the question. Explain only when it is required.

1. Define the center of a character. What can we say about the center of an irreducible character?

2. $\sum_{\chi \in \text{Irr } G} \sum_{g, h \in G} \chi(g)\chi(h) = ?$

3. Consider the character of S_5 which is defined by the action on the partitions of type $3 + 2$. What is the number of the irreducible summands of the restriction of this character to A_5 ?

4. Suppose that $H \leq G$, $\chi \in \text{Irr } H$, and ρ is the regular character of G . Determine the scalar product $[\chi^G, \rho]$.

5. Define the modular character corresponding to a complex character.

6. Which of the following are always algebraic integers if χ is an irreducible character?

A) $\overline{\chi(g)}$ B) $\frac{|\mathcal{K}(g)|}{\chi(1)}$ C) $\frac{|\mathcal{K}(g)|\chi(g)}{\chi(1)}$ D) $\frac{\chi(1)}{\chi(g)}$

7. What can be the number of irreducible characters of a group of order 55?

8. State the Clifford Theorem.

Part II

9. Prove that a simple group cannot have a nontrivial conjugacy class of prime power size. (6 *point*)
10. Prove that the Frobenius kernel is a normal subgroup. (20 *point*)