

Linear transformations, eigen-values, eigen-vectors
Mathematics A2
8th week

1. Which of the following are matrices are orthogonal matrices?

$$\text{a.) } \mathbf{A} = \begin{bmatrix} 2/3 & -2/3 & 1/3 \\ 2/3 & 1/3 & -2/3 \\ 1/3 & 2/3 & 2/3 \end{bmatrix}, \quad \text{b.) } \mathbf{B} = \begin{bmatrix} 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ -1 & 0 & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

2. Find the matrix of the linear transformation $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ that assigns to every vector in \mathbb{R}^2 its

- a.) projection onto the line $y = -\sqrt{3} \cdot x$;
- b.) rotation about the origin by 60° clockwise.

3. Find the matrix of the linear transformation $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ that assigns to every vector in \mathbb{R}^3 its reflection about the plane $x + y = 0$.

4. Find the eigen-values and eigen-vectors for $\mathbf{A} = \begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$.

5. Given the matrix $\mathbf{A} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$,

- a.) Find the matrix that orthogonally diagonalizes \mathbf{A} .
- b.) Give the diagonalized form of the matrix \mathbf{A} .
- c.) Find \mathbf{A}^{10} .