Math A2, Linear Algebre, Sample Test 2.

1. Below you can see the coefficient matrix and the augmented coefficient matrix of a system of linear equations. Find the rank of both, and based on that decide, whether there is a solution or if yes, then how many solutions are. In this case, give the solution (solution set) as well.

$$\begin{pmatrix} 1 & 1 & 2 & |9 \\ 2 & 4 & -3 & |1 \\ 3 & 6 & -5 & |0 \end{pmatrix}$$

2. Check whether the vectors $\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3$ are linearly independent. If yes, then apply the Gram–Schmidt orthogonalization process to them so that to obtain a complete orthonormal basis $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ in $\mathbb{R}^3$: $\mathbf{u}_1 = (0, 3, 4)$, $\mathbf{u}_2 = (1, 2, 2)$, $\mathbf{u}_3 = (1, -1, 0)$.

3. Write the vector $\mathbf{v} = (3, 0, 1)$ as the linear combination of the vectors $\mathbf{u}_1 = (0, 3, 4)$, $\mathbf{u}_2 = (1, 2, 2)$, $\mathbf{u}_3 = (1, -1, 0)$.

4. Check whether the vectors $\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4$ are linearly independent. If yes, then apply the Gram–Schmidt orthogonalization process to them so that to obtain a complete orthonormal basis $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$ in $\mathbb{R}^4$:

   $\mathbf{u}_1 = (0, 2, 1, 0)$, $\mathbf{u}_2 = (1, -1, 0, 0)$, $\mathbf{u}_3 = (1, 2, 0, -1)$, $\mathbf{u}_3 = (1, 0, 0, 1)$.

5. Write the vector $\mathbf{v} = (3, 1, 0, -2)$ as the linear combination of the vectors $\mathbf{u}_1 = (0, 2, 1, 0)$, $\mathbf{u}_2 = (1, -1, 0, 0)$, $\mathbf{u}_3 = (1, 2, 0, -1)$, $\mathbf{u}_3 = (1, 0, 0, 1)$.

6. Find the distance and the angle between the vectors $\mathbf{u} = (2, 2, -1)$ and $\mathbf{v} = (0, 3, 4)$.