

Math. A2 Sample Final, 2016. BSc English
(Linear Algebra)

1. (a) Notion of the determinant. How can you calculate the determinant of a 3×3 matrix?
(b) Notion of an eigenvalue and eigenvector. What do you know about the eigenvalues and eigenvectors of real symmetric matrices?
(c) Can a set of 2 vectors be linearly independent in \mathbb{R}^3 ? And that of 4 vectors? If yes, give an example.
2. For which values of c will the following system of linear equations be consistent? Solve the system with this c :

$$\begin{aligned}x - 2y + 3z - 4w &= 4 \\y - z + w &= -3 \\x + 3y - 3w &= c \\-7y + 3z + w &= -3\end{aligned}$$

3. Let

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix} \quad \text{and} \quad \mathbf{B} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 2 \\ 0 & 2 & 4 \end{pmatrix}$$

be two matrices.

- (a) Find the eigenvalues and eigenvectors of them.
 - (b) Can you diagonalize them? If yes, then find the invertible matrix \mathbf{P} such that $\mathbf{P}^{-1}\mathbf{A}\mathbf{P}$ or $\mathbf{P}^{-1}\mathbf{B}\mathbf{P}$ becomes diagonal.
 - (c) Find the inverse of the matrix \mathbf{A} and \mathbf{B} if exists.
4. The following equation defines a quadratic surface:

$$3x_1^2 + 3x_2^2 + 5x_3^2 - 4x_1x_2 = 1.$$

Find the eigenvalues of the matrix defining the above quadratic form and characterize the surface by them.