Solve the following differential equations

Exact:

1. $e^y + (xe^y - 2y)y' = 0$ 3. $2x^3 - xy^2 + (2y^3 - x^2y)y' = 0$ 5. $\frac{x}{x^2+y^2}\,dx + (\frac{y}{x^2+y^2} - 1)\,dy = 0$

7. $1 + x\sqrt{x^2+y^2} + (y\sqrt{x^2+y^2} - y)y' = 0$ 9. $\ln(y^2+1)\,dx + \frac{2y(x-1)}{y^2+1}\,dy = 0$

Separable:

11. $xy' + y = y^2$ 13. $xy\,dx + \sqrt{1-x^2}\,dy = 0$

15. $\sqrt{1-y^2} = \sqrt{1+x^2}\,y'$ 17. $y'\sin x = y\ln y, \quad y(0) = 1$

By means of integrating factor:

19. $2x^2\sin y\,dx + x^3\cos y\,dy = 0$ 21. $\frac{y}{x}\,dx + (2y^2 - \ln x)\,dy = 0$ 23. $y(1+xy)\,dx - x\,dy = 0$

25. $e^x\cos x - \sin y + y'\cos y = 0$ 27. $y\,dx + (ye^x - 1)\,dy = 0$

Homogeneous (in the variables), substitute $u = y/x$:

29. $(3xy - 2x^2)y' + xy - 2y^2 = 0$ 31. $xe^{y/x} + y = xy'$

33. $y = xy' + \sqrt{x^2+y^2}$ 35. $xy' = y - x\cos^2(y/x)$

First order linear:

37. $y' = xy + x^3, \quad y(0) = 1$ 39. $y' - \frac{2}{x}y = x^2e^x$ 41. $y' + y\tan x = \sin 2x$

43. $y' + y = \sin 2x$ 45. $y'\sin x - y\cos x = e^x\sin^2 x$

Special second order:

55. $xy'' = y'$ 57. $y'' = \frac{y'}{x} + x$ 59. $2xy'y'' = (y')^2 + 1$

61. $4y'' - y = 0$ 63. $y'' = \frac{1}{4\sqrt{y}}$ 65. $y'' = 2yy'$

67. $2y'' = 3y^2, \quad y(-2) = y'(-2) = 1$ 69. $x^2y'' = 2xy' - 3, \quad y(1) = 4, \quad y'(1) = 3$

Second order linear, where y_1 is a solution of the homogeneous part:

71. $x^2(\ln x - 1)y'' - xy' + y = (x\ln x - x)^2, \quad y_1 = x$
73. $y'' + 4y = \frac{8}{\cos 2x}, \quad y_1 = \sin 2x$
75. $2(x+1)^2y'' - (x+1)y' + y = x, \quad y_1 = \sqrt{x+1}$

Second order linear, with constant coefficients:

77. $y'' - 7y' + 10y = 0$ 79. $y'' - 6y' = 0$ 81. $y'' + 6y' + 9y = 0$ 83. $y'' - 2y' + 2y = 0$

85. $y'' + y = 0$ 87. $y'' - 2y' - 3y = 2\cos 3x$ 89. $y'' + y = \tan x$

91. $y'' + y = -4\cos x$ 93. $y'' - y' - 2y = 8e^{3x}$ 95. $y'' - 3y' - 4y = e^{-x}$

97. $y'' - 3y' + 2y = 2e^{x}\cos\dfrac{x}{2}$ 99. $y'' - 9y = 3\sinh 3x$

Apply the known existence and uniquness theorems to the initial value (Cauchy) problems 17 and 37.

Sketch the direction fields and the phase diagrams of the following autonomous differential equations. Characterize the equilibrium solutions of them (stable,unstable,semistable):

a. $y' = (y+1)(y-2)(y-4)$ b. $y' = (y+1)^2(y-2)(y-3)$ c. $y' = (y+1)^2(y-2)^2$ d. $y' = e^{y}-1$