Exercises to the Central Limit theorem topic

1. An astronomer is interested in measuring, in light years, the distance from his observatory to a distant star. Although the astronomer has a measuring technique, he knows that, because of changing atmospheric conditions and normal error, each time a measurement is made it will not yield the exact distance but merely an estimate. As a result the astronomer plans to make a series of measurements and then use the average value of these measurements as his estimated value of the actual distance. If the astronomer believes that the values of the measurements are independent and identically distributed random variables having a common mean d (the actual distance) and a common variance of 4 light years, how many measurements need he make to be reasonably sure that his estimated distance is accurate to within ± 0.5 light years?

2. The number of students that enroll in a psychology course is a Poisson random variable with mean 100. The professor in charge of the course has decided that if the number enrolling is 120 or more he will teach the course in two separate sections, whereas if less than 120 students enroll he will teach all of the students together in a single section. What is the probability that the professor will have to teach two sections?

3. If 10 fair dice are rolled, find the approximate probability that the sum obtained is between 30 and 40.

4. Let $X_i, i = 1, \ldots, 10$ be independent random variables, each uniformly distributed over $(0, 1)$. Calculate an approximation to $P\left\{\sum_{i=1}^{10} X_i > 6\right\}$.

5. Suppose that X is a random variable with mean and variance both equal to 20. What can be said about $P\{0 \leq X \leq 40\}$?

6. From past experience a professor knows that the test score of a student taking his final examination is a random variable with mean 75.

   (a) Give an upper bound to the probability that a student's test score will exceed 85.
   Suppose in addition the professor knows that the variance of a student's test score is equal to 25.

   (b) What can be said about the probability that a student will score between 65 and 85?

(c) How many students would have to take the examination so as to ensure, with probability at least 0.9, that the class average would be within 5 of 75. Do not use the central limit theorem.

7. Use the central limit theorem to solve part (c) of the Previous Problem.

8. Let $X_1, \ldots, X_{20}$ be independent Poisson random variables with mean 1.

(a) Use the Markov inequality to obtain a bound on $P\left\{\sum_{i=1}^{20} X_i > 15\right\}$.

(b) Use the central limit theorem to approximate $P\left\{\sum_{i=1}^{20} X_i > 15\right\}$. For the solutions see the chapters of he book uploaded on my homepage under ceu.