

NAME:

FACULTY:

Math. A3 Final SAMPLE (2008)

1. Solve the following initial value problem: $xy' + 2y = 4x^2$, $y(1) = 2$.
2. Give the equilibrium solutions of the following autonomous differential equation. Then sketch the phase diagram and characterize the equilibrium solutions of it (stable, unstable, semistable): $y' = y^4 + 7y^3 - 30y^2$.
3. Solve the following initial value problem: $y'' + y' - 12y = 0$, $y(2) = 2$, $y'(2) = 0$.
4. Let A and B be independent events with respective probabilities 0.4 and 0.7. Find the probability that exactly one of them happens.
5. In a shop, there appears a * with probability 0.1 on a bill. If a customer gets such a bill (with *) on all the 5 work-days of the week, he/she wins 100 dollars. Through four weeks Fred goes to this shop once a work-day, and buys something. Give the probability that he wins at least 100 dollars.
6. The number of weekly accidents in a large city follows Poisson distribution with parameter 3. Give the probability that there will be exactly 2 accidents within 2 weeks.
7. The distribution of X is given by the density function

$$f(x) = 2x, \quad \text{if } 0 < x < 1,$$

and 0 otherwise.

$$P(X \leq 0.5 | X \geq 0.25) = ?$$

Find the expectation and variance of X !

8. There are 100 students registered for an overall course, but each of them attends the lectures with probability 0.8, independently. What size of a class (with how many chairs) to reserve if we want to give only 5 percent chance to the situation that a student, arriving to the class, cannot find a chair to sit on.
9. Solve the following system of differential equations:

$$x' + 2y' - 3x + 4y = 2 \sin t, \quad 2x' + y' + 2x - y = \cos t$$