List of formulas for the second A3 midterm

Mathematics A3 in English for Civil Engineering students

Solution of $\underline{\dot{x}} = \underline{\underline{A}} \cdot \underline{x}$:

If $\underline{v}^{(j)}$ is an eigenvector with an eigenvalue $r^{(j)}$ of multiplicity one, then the corresponding part of the homogeneous solution is $C_j \cdot \mathrm{e}^{r^{(j)}t} \cdot \underline{v}^{(j)}$.

If $\underline{a} \pm i \cdot \underline{b}$ are eigenvectors with eigenvalues $\lambda \pm i \cdot \mu$ of multiplicity one, then the corresponding part of the homogeneous solution is
$C_j \cdot \mathrm{e}^{\lambda t} \cdot [\underline{a}\cos(\mu t) - \underline{b}\sin(\mu t)] + C_{j+1} \cdot \mathrm{e}^{\lambda t} \cdot [\underline{a}\sin(\mu t) + \underline{b}\cos(\mu t)]$.

For $\underline{\dot{x}} = \underline{\underline{A}} \cdot \underline{x} + \underline{g}(t)$, make the constants C_j t-dependent.

Multiplication rule: $\mathbb{P}\{E_1E_2\dots E_n\} = \mathbb{P}\{E_n \mid E_1\dots E_{n-1}\}\cdots\mathbb{P}\{E_3 \mid E_1E_2\}\cdot\mathbb{P}\{E_2 \mid E_1\}\cdot\mathbb{P}\{E_1\}$,

Law of Total Probability: If $F_1, F_2, \dots$ form a complete system of events that is, $\bigcup_i F_i = S$, and $F_i \cap F_j = \emptyset$ if $i \neq j$, then
$\mathbb{P}\{E\} = \sum_i \mathbb{P}\{E \mid F_i\} \cdot \mathbb{P}\{F_i\}$.

Bayes Theorem: If $F_1, F_2, \dots$ form a complete system of events that is, $\bigcup_i F_i = S$, and $F_i \cap F_j = \emptyset$ if $i \neq j$, then

$$\mathbb{P}\{F_i \mid E\} = \frac{\mathbb{P}\{E \mid F_i\} \cdot \mathbb{P}\{F_i\}}{\sum_j \mathbb{P}\{E \mid F_j\} \cdot \mathbb{P}\{F_j\}}.$$

The binomial(n, p) distr'n

→ mass function: $p(i) = \binom{n}{i} \cdot p^i \cdot (1-p)^{n-i}, \qquad i = 0, 1, \dots, n$

→ expectation: $\mathbb{E}(X) = np$

→ variance: $\mathbf{Var}(X) = np(1-p)$

→ most probable value: $\lfloor (n+1)p \rfloor$

The Poisson(λ) distr'n

→ mass function: $p(i) = \frac{\lambda^i}{i!} \cdot \mathrm{e}^{-\lambda}, \qquad i = 0, 1, 2, \dots$

→ expectation: $\mathbb{E}(X) = \lambda$

→ variance: $\mathbf{Var}(X) = \lambda$

→ most probable value: $\begin{cases} \lfloor \lambda \rfloor & \text{, if } \lambda \text{ is not integer,} \\ \lambda \text{ and } \lambda - 1 & \text{, if } \lambda \text{ is integer.} \end{cases}$

The geometric(p) distr'n

→ mass function: $p(i) = (1-p)^{i-1} \cdot p, \qquad i = 1, 2, 3, \dots$

→ expectation: $\mathbb{E}(X) = \frac{1}{p}$

→ variance: $\mathbf{Var}(X) = \frac{1-p}{p^2}$