

PROBABILITY, Problems to Lesson 3.

1. I have N balls, M red and $N - M$ white, mixed in an urn. n balls are selected randomly without replacement (or at once). Suppose that $n \leq \min\{M, N - M\}$. What is the probability that among the selected n balls there are k red ones ($k = 0, 1, \dots, n$).
2. I have N balls, M red and $N - M$ white, mixed in an urn. n balls are selected randomly with replacement. What is the probability that among the selected (visited) n balls there are k red ones ($k = 0, 1, \dots, n$).
3. What is the probability that by a 5-lottery ticket I win a prize (I have at least a 2-hit)? (5 numbers are chosen out of $1, 2, \dots, 90$)
4. What is the probability that by a 6-lottery ticket I win a prize (I have at least a 3-hit)? (6 numbers are chosen out of $1, 2, \dots, 45$)
5. In a class of 20 students 8 are not prepared. The teacher selects randomly 5 students and asks them. Give the distribution of the number of students who are not able to answer the teacher's question among the selected 5.
6. In a class of 20 students 3 are not prepared. The teacher selects randomly 5 students and asks them. Give the distribution of the number of students who are not able to answer the teacher's question among the selected 5.
7. What is the probability that I have a k -hit by filling in a TOTO ticket randomly ($k = 0, 1, \dots, 13$)? (bet 1, 2, or x on the outcome of each of 13 soccer matches)
8. Give the distribution of the number of girls in a family having n children. Give the mode of this random variable! (Equivalent problem: n fair coins are tossed, or a fair coin is tossed n times; give the distribution of the number of heads.)
9. *Waiting for the first boy.* Consider the following population model: each family waits for a boy, and once they have him, they do not want more children. Give the boys/girls proportion in this population.
10. **Cupon collecting problem.** One of n different kinds of cupons is to be found in each package of a certain washing powder (think of n different color pictures, e.g., red, white, and green, if $n = 3$). If I have a complete collection (at least one of each kind) I can send it to the given address and get a present. On average, how many packages of this washing powder shall I buy, to have a complete collection?
11. *It was enough of cupon collecting.* Under the conditions of the previous exercise, I stop collecting the cupons (buying more washing powder), if I first revisit the same kind of cupon I have already found. Let X denote the number of packages of washing powder I have bought up to the moment, when I decide not to buy more. Give the distribution of X and give an asymptotic to its expectation, if n is large.
12. Cakes are made in a big bakery: the raisins are mixed into the mass and after the cakes are formed randomly. About how many raisins have to be planned for a cake, if they want to make the probability of possible complaints (of not having any raisin in the cake) as small as 0.01. Give the mode of the number of raisins in a cake!