POBABILITY, Lesson 6.

- **Useful Inequalities**

  1. *Markov's Inequality:* Let X be a r.v. taking on nonnegative values. Then

  $$\mathbb{P}(X \geq c) \leq \frac{\mathbb{E}(X)}{c}, \qquad \forall c > 0.$$

  2. *Chebyshev's Inequality:* Let X be a r.v. with finite second moment. Then

  $$\mathbb{P}(|X - \mathbb{E}(X)| \geq \varepsilon) \leq \frac{\mathbb{D}^2(X)}{\varepsilon^2}, \qquad \forall \varepsilon > 0.$$

  3. *Chernoff's Inequality:* $X_1, \ldots, X_n$ i.i.d., $|X_i| \leq K$, $X := \sum_{i=1}^n X_i$. Then

  $$\mathbb{P}(|X - \mathbb{E}(X)| \geq a) \leq e^{-\frac{a^2}{2(\mathbb{D}^2(X) + Ka/3)}}, \qquad \forall a > 0.$$

- **Laws of Large Numbers, Central Limit Theorem**

  1. *Weak Law*: If $X_1, \ldots, X_n$ are i.i.d. with finite $\mathbb{E}(X_i) = \mu$, then $\overline{X}_n \to \mu$ in probability:

  $$\lim_{n\to\infty} \mathbb{P}(|\overline{X}_n - \mu| > \varepsilon) = 0, \quad \forall \varepsilon > 0, \text{ where } \overline{X}_n = \frac{1}{n}\sum_{i=1}^n X_i.$$

  2. *Strong Law*: If $X_1, \ldots, X_n$ are i.i.d. with finite $\mathbb{E}(X_i) = \mu$, then $\overline{X}_n \to \mu$ almost surely:

  $$\mathbb{P}(\lim_{n\to\infty} \overline{X}_n = \mu) = 1.$$

  3. **CLT**: If $X_1, \ldots, X_n$ are i.i.d. with finite $\mathbb{E}(X_i) = \mu$ and $\mathbb{D}^2(X_i) = \sigma^2$, then

  $$\frac{\sum_{i=1}^n X_i - n\mu}{\sqrt{n}\sigma} \to \mathcal{N}(0,1) \quad \text{in distribution (convergence of c.d.f.'s)}, \quad n \to \infty.$$