PROBABILITY A4, Problems to Lesson 1.

1. How many different letter arrangements can be formed using the letters ABC? PEPPER?

2. A committee of 3 is delegated from a group of 13 people. How many different committees are possible? How many different committees are possible, if the delegated 3 represent different posts (president, secretary, cashier)?

3. How many different licence plates (3 letters – 3 digits) can be formed? (there are 26 letters and 10 digits available)

4. How many 3-digit numbers with decreasing digits can be made? (**BONUS**)

5. Two teams of equal size are formed randomly out of 22 football players. What is the probability that the two best players are in one team?

6. How many different ways a TOTO ticket can be filled in? (bet 1, 2, or x on the outcome of each of 13 soccer matches)

7. How many different ways a LOTTO (lottery) ticket can be filled in? (5 numbers are chosen out of 1,2,...,90)

8. How many different parties in a group of n people can be formed? (any number of people can form a party)

9. In a pastry shop there are 13 different kinds of cake available. How many different ways can we choose 3 different pieces of cake? How many different ways can we choose 3 pieces of cake? How many different ways can we take home 20 pieces of cake?

10. What is the probability (p_k) that in a group of k people there are at least two celebrating his/her birthday on the same day? Surprisingly, $p_{23} \approx 0.5$ and $p_{55} \approx 0.99$ (birthday paradox).

11. How many different ways n different (color) balls can be put into N urns? (Equivalent problem: How many different ways n different objects can be distributed among N people?) What is the probability that there are exactly k balls in a certain urn? (Maxwell–Bolzmann statistic for spin states of gas molecules.)

12. How many different ways n (indistinguishable) particles can exist in N different states? (Equivalent problem: How many different ways n "100 Ft" coins can be distributed among N people?) (Bose–Einstein statistic for photons.)

13. There are n criminals in a prison. They may be set free in the following way: the warden writes their numbers separately to n pieces of paper, then picks n of them, one after the other, with replacement. The ones whose number came out are set free. What is the probability that all of them are set free; the ringleader is set free; the ringleader and his best friend are set free? Investigate the results as $n \to \infty$. Simulate the $n = 6$ case by casting a die 6 times (the ones, whose number comes out, are set free).

14. A DNA-sequence consists of letters A,T,C,G. How many different 20-length sequences containing all of the four letters can be formed? (Equivalent problem: each of 20 students buys an icecream, one of the flavors A,T,C,G. How many different ways can it be realized, if all the four flavors are tested by at least one student?) (**BONUS**)