

PROBABILITY A4, Problems to Lessons 10-11.

1. Consider the following joint densities of X and Y :

(a)

$$f(x, y) = \frac{2x}{y} \quad (0 < x < 1, x < y < \frac{1}{x})$$

(b)

$$f(x, y) = \begin{cases} e^{x+y} & \text{if } x > 0, y > 0, \\ 0 & \text{otherwise,} \end{cases}$$

(c) (X, Y) is uniformly distributed on the unit circle.

- Calculate the marginal distributions! Are X and Y independent?
- Find the p.d.f. of the conditional distribution of Y conditioned on $X = x$! Find the $\mathbb{E}(Y|X = x)$ conditional expectation!
- Find $\text{Cov}(X, Y)$ and $\text{Corr}(X, Y)$!

2. Calculate the p.d.f. of the following random variable: if X is uniform on $[-2, 4]$, then find the p.d.f. of $Y := X^2$.

3. The BME management claims that the average score of students admitted to the Faculty of Electric Engineering was 430 points. We have the data for 10 students: 390, 400, 410, 395, 480, 485, 450, 430, 400, 410.

(a) Assuming that the data is normally distributed with a standard deviation of 5 points, give a 95% confidence interval for the claim. Shall we accept the hypothesis at this confidence level? Explain in words what your decision means.

(b) Assuming that the data is normally distributed with unknown standard deviation, give a 95% confidence interval for the claim. Shall we accept the hypothesis at this confidence level? Explain in words what your decision means.

4. Let X_1, \dots, X_n be i.i.d. sample from the distribution with p.d.f. $f_\theta(x) = 2\sqrt{\frac{\theta}{\pi}}e^{-\theta x^2}$ (if $x \geq 0$, otherwise 0). Find the ML-estimate of $\theta > 0$.

5. Let (X, Y) have bivariate normal distribution with expectation vector $\boldsymbol{\mu} = (1, 2)$ and covariance matrix

$$\mathbf{C} = \begin{pmatrix} 2 & -1 \\ -1 & 3 \end{pmatrix}.$$

Find the distribution of $X, Y, X + Y, X - Y, 8X - 4Y$.