PROBABILITY AND STATISTICS, Problems to Lesson 3.

1. We have N balls, M red and $N-M$ white, mixed in an urn. n balls are selected randomly without replacement (or at the same time). Suppose that $n \le \min\{M, N-M\}$. What is the probability that among the selected n balls there are k red ones ($k = 0, 1, \ldots, n$).
2. We have N balls, M red and $N-M$ white, mixed in an urn. n balls are selected randomly with replacement. What is the probability that among the selected (visited) n balls there are k red ones ($k = 0, 1, \ldots, n$).
3. What is the probability that by a 5-lottery ticket one wins a prize (one has at least a 2-hit)? (5 numbers are selected from $\{1, 2, \ldots, 90\}$)
4. What is the probability that by a 6-lottery ticket one wins a prize (one has at least a 3-hit)? (6 numbers are selected from $\{1, 2, \ldots, 45\}$)
5. In a class of 20 students 8 are not prepared for the class. The teacher selects 5 students at random and asks them. Give the distribution of the number of students who are not able to answer the teacher's question among the selected 5.
6. In a class of 20 students 3 are not prepared for the class. The teacher selects 5 students at random and asks them. Give the distribution of the number of students who are not able to answer the teacher's question among the selected 5.
7. What is the probability that we have a k-hit by filling in a TOTO ticket at random ($k = 0, 1, \ldots, 13$)? (bet 1, 2, or x on the outcome of each of 13 soccer matches)
8. Give the distribution of the number of girls in a family having n children. Give the mode of this random variable! (The gender of children is independent of each other with probability 1/2–1/2.) Equivalent problem: n fair coins are tossed, or a fair coin is tossed n times; give the distribution of the number of heads.
9. *Waiting for the first boy.* Consider the following population model: each family waits for a boy, and once they have him, they do not want more children. Give the boys/girls proportion in this population. (The gender of children is independent of each other with probability 1/2–1/2.)
10. *Coupon collecting problem.* One of n different kinds of coupons is to be found in each package of a certain washing powder (think of n different color pictures, e.g., red, white, and green, if $n = 3$). If we have a complete collection (at least one of each kind) we can send it to the given address and get a present. On average, how many packages of this washing powder people buy to have a complete collection? (Give asymptotics as $n \to \infty$.)
11. (**BONUS**) *It was enough of coupon collecting.* Under the conditions of the previous exercise, we stop collecting the coupons (buying more washing powder) if we first revisits the same kind of coupon we have already found. Let X denote the number of packages of washing powder I have bought up to the moment, when we decide not to buy more. Give the distribution of X and give the asymptotic value of its expectation if n is large.
12. Cookies are made in a big bakery: the blueberries are mixed into the mass and then the cakes are formed randomly. About how many blueberries have to be planned for a cookie, if they want to make the probability of possible complaints (of not having any blueberry in a cookie) as small as 0.01. Give the mode of the number of blueberries in a cookie!