Exercises to Notable Distributions

Marianna Bolla, Prof, DSc.
Institute of Mathematics, BME

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1. We have N balls, M red and $N-M$ white, mixed in an urn. n balls are selected randomly without replacement (or at the same time). Suppose that $n \leq \min\{M, N-M\}$. What is the probability that among the selected n balls there are k red ones ($k = 0, 1, \ldots, n$).

2. We have N balls, M red and $N-M$ white, mixed in an urn. n balls are selected randomly with replacement. What is the probability that among the selected (visited) n balls there are k red ones ($k = 0, 1, \ldots, n$).

3. What is the probability that by a 5-lottery ticket one wins a prize (one has at least a 2-hit)? (5 numbers are selected from $\{1, 2, \ldots, 90\}$)

4. In a class of 20 students 8 are not prepared for the class. The teacher selects 5 students at random and asks them. Give the distribution of the number of students who are not able to answer the teacher's question among the selected 5.

5. What is the probability that we have a k-hit by filling in a TOTO ticket at random ($k = 0, 1, \ldots, 13$)? (bet 1, 2, or x on the outcome of each of 13 soccer matches)

6. Give the distribution of the number of girls in a family having n children. Give the mode of this random variable! (The gender of children is independent of each other with probability 1/2–1/2.) Equivalent problem: n fair coins are tossed, or a fair coin is tossed n times; give the distribution of the number of heads.

7. *Waiting for the first boy.* Consider the following population model: each family waits for a boy, and once they have him, they do not want more children. Give the boys/girls proportion in this population. (The

gender of children is independent of each other with probability 1/2–1/2.)

8. Cookies are made in a big bakery: the blueberries are mixed into the mass and then the cakes are formed randomly. About how many blueberries have to be planned for a cookie, if they want to make the probability of possible complaints (of not having any blueberry in the cookie) as small as 0.01. Give the mode of the actual number of blueberries in a cookie!

9. *Birthday holidays:* n workers are employed in a factory. Each working day, each worker manufactures a chair. When any of them has a birthday, there is a holiday (the factory is closed, they do not work at all). Under these conditions, how many workers have to be employed if they want to maximize the average number of chairs produced in a year in the factory? Equivalent problem: A worker's legal code specifies a holiday any day during which at least one worker in a certain factory has a birthday. All other days are working days. How many workers (n) must the factory employ so that the expected number of working man-days is maximized during the year?

10. Let $X \sim \mathcal{N}(\mu, \sigma^2)$ be Gaussian random variable. Calculate $\mathbb{P}(\mu - \sigma < X < \mu + \sigma)$ and $\mathbb{P}(\mu - 2\sigma < X < \mu + 2\sigma)$.