

SMD Homework Exercises 1.

1. There are 100 students registered for an overall course, but each of them attends the lectures with probability 0.8 only, independently of each other. What size of a class (with how many chairs) to reserve if we want to give only 5% chance to the situation that a student, arriving to the class, cannot find a free chair.
2. Let X_1, \dots, X_3 be an i.i.d. sample from Poisson distribution with unknown parameter. Which of the following statistics is an unbiased estimator of λ ? Why?
 - (a) \bar{X}
 - (b) X_1
 - (c) $\frac{1}{2}X_1 + \frac{1}{3}X_2$
 - (d) $\frac{1}{3}X_1 + \frac{1}{3}X_2 + \frac{1}{3}X_3$
 - (e) $\frac{1}{2}X_1 + \frac{1}{3}X_2 + \frac{1}{6}X_3$
 - (f) $\frac{1}{3} \sum_{i=1}^3 (X_i - \bar{X})^2$
 - (g) $\frac{1}{2} \sum_{i=1}^3 (X_i - \bar{X})^2$
 - (h) $\sum_{i=1}^3 (X_i - \bar{X})^2$
3. Prove that the corrected empirical variance is unbiased, whereas the empirical variance is asymptotically unbiased estimator of the population variance if it exists.
4. An old joke is that a certain professor left Princeton for Stanford, and thereby improved the average quality of both departments. Is this possible? Explain your answer!
5. Consider a randomly selected two-children family from a population where the genders of children are independent of each other, but the boy:girl probability at birth may differ a bit from the $\frac{1}{2}:\frac{1}{2}$. Find the probabilities below! How the difference between the two behaves if we get farther and farther from the $\frac{1}{2}:\frac{1}{2}$ ratio?
 - (a) The two children have the same gender.
 - (b) The two children have different genders.

(The same with tossing a not necessarily fair coin.)