SMD Homework Exercises 2. (2024 fall)

1. There are 100 students registered for an overall course, but each of them attends the lectures with probability 0.8 only, independently of each other. What size of a class (with how many chairs) to reserve if we want to give only 5% chance to the situation that a student, arriving to the class, cannot find a free chair.

2. Prove that the corrected empirical variance is unbiased, whereas the empirical variance is asymptotically unbiased estimator of the population variance if it exists.

3. An old joke is that a certain professor left Princeton for Stanford, and thereby improved the average quality of both departments. Is this possible? Explain your answer!

4. Let $x_1, \ldots, x_{20}$ be the realization of an i.i.d. sample from Gaussian distribution with unknown expectation μ and known standard deviation 0.05. Based on $\overline{x} = 1.5$, give the 98% confidence interval for μ.

5. Let $x_1, \ldots, x_{20}$ be the realization of an i.i.d. sample from Gaussian distribution with unknown expectation μ and unknown standard deviation. Based on $\overline{x} = 1.5$ and $s^* = 0.05$ give the 98% confidence interval for μ.

For **BONUS** points:

- *Birthday holidays:* n workers are employed in a factory. Each working day, each worker manufactures a chair. When any of them has a birthday, there is a holiday (the factory is closed, they do not work at all). Under these conditions, how many workers have to be employed if they want to maximize the average number of chairs produced in a year in the factory?

  Equivalent problem: A worker's legal code specifies a holiday any day during which at least one worker in a certain factory has a birthday. All other days are working days. How many workers (n) must the factory employ so that the expected number of working man-days is maximized during the year?

- *Coupon collecting problem:* One of n different kinds of coupons is to be found in each package of a certain washing powder (think of n different color pictures, e.g., red, white, and green, if $n = 3$). If we have a complete collection (at least one of each kind) we can send it to the given address and get a present. On average, how many packages of this washing powder people buy to have a complete collection? (Give asymptotics as $n \to \infty$.)