SMD Homework Exercises 3. (2024 fall)

1. Let $X_1, \ldots, X_3$ be an i.i.d. sample from Poisson distribution with unknown parameter. Which of the following statistics is an unbiased estimator of λ? Why?

   (a) $\overline{X}$

   (b) X_1

   (c) $\frac{1}{2}X_1 + \frac{1}{3}X_2$

   (d) $\frac{1}{3}X_1 + \frac{1}{3}X_2 + \frac{1}{3}X_3$

   (e) $\frac{1}{2}X_1 + \frac{1}{3}X_2 + \frac{1}{6}X_3$

   (f) $\frac{1}{3}\sum_{i=1}^{3}(X_i - \bar{X})^2$

   (g) $\frac{1}{2}\sum_{i=1}^{3}(X_i - \bar{X})^2$

   (h) $\sum_{i=1}^{3}(X_i - \bar{X})^2$

2. R.A. Fisher investigated the effect of two seducers on 10 patients. The surplus sleep, A and B pills cause (in hours) are below:

No.	A	B	B-A
1.	+0.7	+1.9	+1.2
2.	−1.6	+0.8	+2.4
3.	−0.2	+1.1	+1.3
4.	−1.2	+0.1	+1.3
5.	−0.1	−0.1	0.0
6.	+3.4	+4.4	+1.0
7.	+3.7	+5.5	+1.8
8.	+0.8	+1.6	+0.8
9.	0.0	+4.6	+4.6
10.	+2.0	+3.4	+1.4

   Is the difference between the effect of the two seducers significant? If yes, then is seducer B significantly better than A? Decide with different levels of significance! Be careful, which kind of t-test you use!

3. 10-10 nails are manufactured on two machines. The average sizes (cm) and corrected empirical standard deviations are:

$$\bar{x} = 0.625, \quad \bar{y} = 0.471, \quad s_x^* = 0.754, \quad s_y^* = 1.269.$$

   Compare the variances of the production of the two machines with F-test, and investigate the null-hypothesis that there is no difference in the sizes of the production of the two machines! Use $\alpha = 0.10$ for the level of significance! Be careful, which kind of t-test you use!

   How would you test the same hypothesis if 100-100 nails are manufactured on two machines with the same empirical data?

4. Is the chance of hypertony is the same in normal and overweighted population? Decide using the following evidences with $\alpha = 0.01$. Out of 4200 normal patients 792, while out of 1000 overweighted ones 249 suffered of hypertony.

   Next test, whether the overweight increases the chance of hypertony.

5. We want to prove that the percentage of faulty production (θ) is more than 5% in a production. To test the hypothesis

$$H_0 : \ \theta \leq 0.05 \quad \text{versus} \quad H_0 : \ \theta > 0.05$$

we take a 25 element sample out of the production. Let X denote the number of faulty products among the 25 ones. Find the significance (P-value) and sketch the power function of the following three tests based on rejection regions

$$a.\ \{X \geq 2\} \qquad b.\ \{X \geq 3\} \qquad c.\ \{X \geq 4\}.$$