

Let $p > q$ denote the proportions of the black and white murderers sentenced to death when the victim was black, and let $r > s$ denote the same proportions when the victim was white. The paradox can hold with some relative frequencies w_1, w_2 of black murderers killing black and white victims, and with relative frequencies w_3, w_4 of white murderers killing black and white victims, respectively, if

$$w_1p + w_2r < w_3q + w_4s. \tag{1}$$

Prove that there are $0 < w_1, w_2, w_3, w_4 < 1$ real numbers with $w_1 + w_2 = 1$, $w_3 + w_4 = 1$ and satisfying (1) if and only if $q > r$ or $s > p$.