POBABILITY AND STATISTICS, Problems to Lessons 11-12.

1. Let $X_1, \dots, X_n \sim \mathcal{N}(\mu, \sigma_0^2)$ be i.i.d. sample with given variance. E.g., we measure the weight of n packages of sugar powder (in kg). Let $n = 25$, $\sigma_0 = 0.05$, $\bar{x} = 0.98$. We want to decide about

$$H_0 : \mu = 1(kg) \quad \text{versus} \quad H_0 : \mu \neq 1(kg).$$

   Give the decision with 2-tailed u-test, if $\varepsilon = 0.05$ and $\varepsilon = 0.01$.

2. Let x be the realization of a one-element $\mathcal{E}xp(\lambda)$ sample. For the alternative

$$H_0 : \lambda = 1 \quad \text{versus} \quad H_1 : 0 < \lambda < 1$$

   the test with critical region $\{x \,|\, x \geq c\}$ is uniformly most powerful. Give the value c such that the Type I. Error (size) be 0.01! With this c give the power of the test (as a function of λ, when $0 < \lambda < 1$).

3. *Paired sample t-test.* R.A. Fisher investigated the effect of two seducers on 10 patients. The surplus sleep, A and B pills cause (in hours) are below:

No.	A	B	B-A
1.	+0.7	+1.9	+1.2
2.	−1.6	+0.8	+2.4
3.	−0.2	+1.1	+1.3
4.	−1.2	+0.1	+1.3
5.	−0.1	−0.1	0.0
6.	+3.4	+4.4	+1.0
7.	+3.7	+5.5	+1.8
8.	+0.8	+1.6	+0.8
9.	0.0	+4.6	+4.6
10.	+2.0	+3.4	+1.4

   Is the difference between the two seducers significant? Let Z be the difference of the surplus times.

$$H_0 : \mathbb{E}(Z) = 0 \quad \text{vers.} \quad H_1 : \mathbb{E}(Z) \neq 0$$

   $n = 10$, $\bar{z} = 1.58$, $s_{10}^* = 1.23$, hence

$$t = \frac{1.58 - 0}{1.23} \cdot \sqrt{10} = 4.06.$$

6. Decide whether the following *die is fair or not.* We cast it $n = 1200$ times and the frequencies of the sides are: $\nu_1 = 184$, $\nu_2 = 212$, $\nu_3 = 190$, $\nu_4 = 208$, $\nu_5 = 212$, $\nu_6 = 194$.

$$\chi^2 = \sum_{i=1}^{6} \frac{(\nu_i - 200)^2}{200} = \frac{16^2 + 12^2 + 10^2 + 8^2 + 12^2 + 6^2}{200} = 3.72.$$