PROBABILITY AND STATISTICS, Homework Exercises 1.

1. A group of 10 people orders 5 beers, 3 coffees and 2 cakes in a restaurant (each of them orders one item and there is only one flavour of beer, cofee, and cake available). The absent-minded waiter forgets who ordered what, and hence, he randomly gives to ech person an item. What is the probability that everybody gets what he/she had wanted.

2. Classical problem of noble man De Méré: How the probabilities below are related to each other? Compare the numerical values.

   (a) Casting a fair die 4 times, find the probability that at least one of the outcomes is the number 6.

   (b) Casting two fair dice 24 times, find the probability that at least one of the outcomes is a double 6.

3. Consider a randomly selected two-children family from a population where the gender of children is independent of each other, but the boy – girl probability is a little bit differs from the 1/2 – 1/2. How the probabilities below are related to each other? How the difference between the two behaves if we get farther and farther from the 1/2 – 1/2 ratio?

   (a) The two children have the same gender.

   (b) The two children have distinct genders.

4. (a) A red and a green fair dice are rolled. What is the conditional probability that the red one leads to 6 given that the sum of the dice is k? Compute for all possible values of $2 \leq k \leq 12$.

   (b) Two red dice are rolled. Find the probability that the maximum of the two numbers is k (k=1,2,3,4,5,6). Then find the probability that the minimum of the two numbers is k (k=1,2,3,4,5,6).

5. Three cooks, A, B, and C bake a special kind of cake, and with respective probabilities 0.03, 0.05, and 0.06 it fails to rise. In the restaurant where they work, A bakes 50 per cent of these cakes, B 30 per cent, and C 20 per cent. What is the average proportion of failures? What proportion of "failures" is caused by A? by B? by C?