

PROBABILITY AND STATISTICS, Homework Exercises 3.

1. There are 100 students registered for an overall course, but each of them attends the lectures with probability 0.8, independently. What size of a class (with how many chairs) to reserve if we want to give only 5 percent chance to the situation that a student, arriving to the class, cannot find a chair to sit on.
2. An old joke is that a certain professor left Princeton to go to Stanford and thereby improved the average quality of both departments. Is this possible? Explain your answer!
3. Let $X_1^* \leq \dots \leq X_n^*$ be order statistics corresponding to an $\mathcal{Exp}(\lambda)$ i.i.d. sample. Find the expectation and variance of X_k^* ($k = 1, \dots, n$).
4. Let the random variables X and Y have the joint p.d.f.

$$f(x, y) = 2e^{-(x+y)}, \quad \text{if } 0 < x < y.$$

Calculate the probability $\mathbb{P}(Y < 3X)$. Hint: think of the exponential order statistics.

5. Let $Y_1^* \leq \dots \leq Y_n^*$ be order statistics corresponding to a $\mathcal{U}(0, 1)$ i.i.d. sample. Determine the distribution of the sample range $Y_n^* - Y_1^*$.