PROBABILITY AND STATISTICS, Homework Exercises 4.

1. Let $X_1, \dots, X_n \sim \mathcal{G}(\theta)$ be i.i.d. sample.
   (a) On the basis of this sample find a sufficient statistic for the parameter θ of the geometric distribution!
   (b) On the basis of this sample find maximum likelihood estimate of θ!

2. Let $X_1, \dots, X_n$ be i.i.d. sample from the absolute continuous distribution given by the p.d.f.

$$f_\theta(x) = \frac{\theta}{(x+1)^{\theta+1}},$$

   if $x \geq 0$, and 0, otherwise ($\theta > 0$ is parameter).
   (a) Find a sufficient statistic for θ!
   (b) Does the statistic $\prod_{i=1}^n \frac{1}{X_i+1}$ provide an unbiased estimate for the parameter θ?
   (c) On the basis of this sample find maximum likelihood estimate of θ!

3. Let $X_1, \dots, X_n$ be i.i.d. sample from the absolute continuous distribution given by the p.d.f.

$$f_\theta(x) = \frac{4x^3}{\theta^4},$$

   if $0 \leq x \leq \theta$, and 0, otherwise ($\theta > 0$ is parameter).
   (a) Find a sufficient statistic for θ!
   (b) On the basis of this sample find maximum likelihood estimate of θ!

4. Let $X_1, \dots, X_n$ be i.i.d. sample from a Pareto distribution with p.d.f.

$$f_{\alpha,\beta}(x) = \frac{\beta\alpha^\beta}{x^{\beta+1}}, \quad \text{if} \quad x \geq \alpha,$$

   and 0 otherwise. Give maximum likelihood estimates of the parameters $\alpha > 0$, $\beta > 0$ on the basis of the above sample!

5. Let $X_1, \dots, X_n$ be i.i.d. sample, suppose that $E(X_1^2) < \infty$.
   (a) Prove that the empirical variance of the sample provides an asymptotically unbiased estimate for the variance of the underlying distribution!
   (b) Prove that the corrected empirical variance of the sample provides an unbiased estimate for the variance of the underlying distribution!