**MATHEMATICAL STATISTICS, Homework Exercises 1.**

1. An old joke is that a certain professor left Princeton to go to Stanford and thereby improved the average quality of both departments. Is this possible? Explain your answer!

2. Let $X_1^* \le \dots \le X_n^*$ be order statistics corresponding to an $\mathcal{E}xp(\lambda)$ i.i.d. sample. Find the p.d.f. and c.d.f. of $X_k^*$ $(k = 1, \dots, n)$.

3. Let $X_1^* \le \dots \le X_n^*$ be order statistics corresponding to an $\mathcal{E}xp(\lambda)$ i.i.d. sample. Let $U_1 = X_1^*$ and $U_k = X_k^* - X_{k-1}^*$ $(k = 2, \dots, n)$ be the differences between the consecutive ones. Find the joint and the individual distributions of $U_1, U_2, \dots, U_n$.

4. Let $X_1, \dots, X_n \sim \mathcal{G}(\theta)$ be i.i.d. sample.
   (a) On the basis of this sample find a sufficient statistic for the parameter $\theta$ of the geometric distribution!
   (b) On the basis of this sample find the maximum likelihood estimator of $\theta$!

5. Let $X_1, \dots, X_n$ be i.i.d. sample from the absolute continuous distribution given by the p.d.f.
$$f_\theta(x) = \frac{\theta}{(x+1)^{\theta+1}},$$
if $x \ge 0$, and 0, otherwise ($\theta > 0$ is parameter).
   (a) Find a sufficient statistic for $\theta$!
   (b) Is the statistic $\prod_{i=1}^n \frac{1}{X_i+1}$ an unbiased estimator for the parameter $\theta$?
   (c) On the basis of this sample find the maximum likelihood estimator of $\theta$!

6. Let $X_1, \dots, X_n$ be i.i.d. sample from the absolute continuous distribution given by the p.d.f.
$$f_\theta(x) = \frac{4x^3}{\theta^4},$$
if $0 \le x \le \theta$, and 0, otherwise ($\theta > 0$ is parameter).
   (a) Find a sufficient statistic for $\theta$!
   (b) On the basis of this sample find the maximum likelihood estimator of $\theta$!

7. Let $X_1, \dots, X_n$ be i.i.d. sample from a Pareto distribution with p.d.f.
$$f_{\alpha,\beta}(x) = \frac{\beta\alpha^\beta}{x^{\beta+1}}, \quad \text{if} \quad x \ge \alpha,$$
and 0 otherwise. Give the maximum likelihood estimator of the parameters $\alpha > 0$, $\beta > 0$ on the basis of the above sample!

8. Let $X_1, \dots, X_n$ be i.i.d. sample, suppose that $E(X_1^2) < \infty$.
   (a) Prove that the sample variance is an asymptotically unbiased estimator for the true variance of the underlying distribution!
   (b) Prove that the corrected sample variance is an unbiased estimator for the true variance of the underlying distribution!