MATHEMATICAL STATISTICS, Homework Exercises 2.

1. Test the hypothesis whether the following die is fair with level of significance $\alpha = 0.05$. We cast it $n = 1200$ times and get the frequencies of the sides: $\nu_1 = 184$, $\nu_2 = 212$, $\nu_3 = 190$, $\nu_4 = 208$, $\nu_5 = 212$, $\nu_6 = 194$.

2. We asked 460 persons whether they like coffee (yes/no) or tea (yes/no). 416 like both, 5 like none of them, 16 like coffee but do not like tea, and 23 like tea but do not like coffee. Decide whether the attitudes toward coffee and tea are independent with level of significance $\alpha = 0.05$.

3. The number of boys in 500 families with 5 children is investigated. There were 20 families with no boy, 75 with 1, 145 with 2, 140 with 3, 85 with 4, and 35 with 5 boys. Decide (with level of significance $\alpha = 0.05$) whether the number of boys in a 5-children family follows binomial distribution.

4. The following scores are obtained on a test of dexterity and aggression administered to a random sample of 10 high-school seniors:

Dexterity	23	29	45	36	49	41	30	15	42	38
Aggression	45	48	16	28	38	21	36	18	31	37

   Find the Spearman's rank correlation between dexterity and aggression based on this sample.

5. We want to prove that the percentage of waste (θ) is more than 5% in a production. To test the hypothesis

$$H_0: \ \theta \leq 0.05 \quad \text{versus} \quad H_0: \ \theta > 0.05$$

   we take a 25 element sample out of the production. Let X denote the number of waste products among the 25 ones. Find the significance and sketch the power function of the following three tests based on critical regions

$$a.\ \{X \geq 2\} \qquad b.\ \{X \geq 3\} \qquad c.\ \{X \geq 4\}.$$

6. 10-10 nails are manufactured on two machines. The average sizes (cm) and corrected empirical standard deviations are:

$$\bar{x} = 0.625, \quad \bar{y} = 0.471, \quad s_x^* = 0.754, \quad s_y^* = 1.269.$$

   Compare the variances of the production of the two machines with F-test, and investigate the null-hypothesis that there is no difference in the sizes of the production of the two machines! Use $\alpha = 0.10$ for the level of significance! Be careful, which kind of t-test you use! How would you test the same hypothesis if 100-100 nails are manufactured on two machines with the same empirical data?

7. Is the chance of hypertony is the same in normal and overweighted population? Decide it using the following evidences with $\alpha = 0.01$. Out of 4200 normal patients 792, while out of 1000 overweighted ones 249 suffer of hypertony. Next decide, whether the overweight increases the chance of hypertony, significantly.

8. Let $X_1, \ldots, X_n$ be i.i.d. sample from Cauchy distribution with parameter θ. Its p.d.f. is

$$f_\theta(x) = \frac{1}{\pi} \frac{1}{1 + (x - \theta)^2}.$$

Find the Fisher information content of the above sample!

9. Let $X_1, \ldots, X_n$ be i.i.d. sample from Poisson distribution with parameter λ. Construct an unbiased estimator for the parameter function $\psi(\lambda) = \lambda^2 e^{-\lambda}$ with the help of X_1 only. Then blackwellize it with a sufficient statistic!

10. Let $X_1, \ldots, X_n \sim \Gamma_\alpha(\lambda)$ be i.i.d. sample, where $\alpha > 0$, $\lambda > 0$ are unknown parameters. Give moment estimator for the parameter (α, λ) based on the above i.i.d. sample!