STATISTICS, Practice Exercises

1. Let $X_1, \ldots, X_n \sim \mathcal{G}(\theta)$ be i.i.d. sample.

   (a) On the basis of this sample find a sufficient statistic for the parameter θ of the geometric distribution!

   (b) On the basis of this sample find maximum likelihood (ML) estimate of θ!

2. Let $X_1, \ldots, X_n$ be i.i.d. sample from the absolute continuous distribution given by the p.d.f.

$$f_\theta(x) = \frac{4x^3}{\theta^4},$$

   if $0 \leq x \leq \theta$, and 0, otherwise ($\theta > 0$ is parameter).

   (a) Find a sufficient statistic for θ!

   (b) On the basis of this sample find maximum likelihood (ML) estimate of θ!

3. R.A. Fisher investigated the effect of two seducers on 10 patients. The surplus sleep, A and B pills cause (in hours) are below:

No.	A	B	B-A
1.	+0.7	+1.9	+1.2
2.	−1.6	+0.8	+2.4
3.	−0.2	+1.1	+1.3
4.	−1.2	+0.1	+1.3
5.	−0.1	−0.1	0.0
6.	+3.4	+4.4	+1.0
7.	+3.7	+5.5	+1.8
8.	+0.8	+1.6	+0.8
9.	0.0	+4.6	+4.6
10.	+2.0	+3.4	+1.4

   Is the difference between the effect of the two seducers significant? If yes, then is seducer B significantly better than A? Decide with different levels of significance! Be careful, which kind of t-test you use!

4. Is the chance of hypertony is the same in normal and overweighted population? Decide using the following evidences with $\alpha = 0.01$. Out of 4200 normal patients 792, while out of 1000 overweighted ones 249 suffered of hypertony. Next decide, whether the overweight increases the chance of hypertony.

5. Decide whether the following *die is fair or not*. We cast it $n = 1200$ times and the frequencies of the sides are: $\nu_1 = 184$, $\nu_2 = 212$, $\nu_3 = 190$, $\nu_4 = 208$, $\nu_5 = 212$, $\nu_6 = 194$.

$$\chi^2 = \sum_{i=1}^{6} \frac{(\nu_i - 200)^2}{200} = \frac{16^2 + 12^2 + 10^2 + 8^2 + 12^2 + 6^2}{200} = 3.72.$$

6. 90%-os szinten vizsgálja meg, hogy az alábbi 100 elemű minta származhat-e

(a) 3 paraméterű Poisson eloszlásból? A mintában a 0,1,2,3,4 értékek fordultak elő a következő gyakoriságokkal:

$$12, 32, 25, 21, 10.$$

(Használja a Poisson- és χ^2-eloszlások táblázatát!)

(b) Egyáltalán származhat-e a minta Poisson eloszlásból?