SPECTRAL CLUSTERING

Midterm assignments (2020 fall semester)

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You are assumed to choose between the following midterm assignments (each of them is for 50 points maximum, and you must have minimum 20 for the signature):

- Solve at least 4 of the theoretical Exercises below. Computer aided results are not accepted, you have to turn in detailed calculations in handwriting.
- Find a paper (there are plenty after the lessons) and prepare a 2-3 pages long description about the most important issues, theorems in it, possibly with proof. Also give a 10 minutes presentation. You can choose a shorter paper or part of a longer one. You can work in pairs as well, but the collaborating students are supposed to work on disjoint parts of a paper. Other papers are also welcome with some math content.
- Illustrate a spectral clustering method on a real-life dataset, and explain your results in a 10 minutes presentation. Also prepare a 2-3 pages description. You can find data on the homepage of M. E. J. Newman:
  www-personal.umich.edu/∼mejn/netdata
  but you can use any other network data.

Please, write me (in e-mail) until the end of October, which option you choose, and in case of presentation, please give me the paper title you select. Then you are supposed to send me the solutions (if you solve exercises) or the writeup of presentation or data processing before 15 November, so that I can make a schedule of talks. The last week is for a written exam, the result of which can be improved by an oral one in the exam period.

Theoretical exercises

1. Find the adjacency eigenvalues of the complete graph K_n and of the complete bipartite graph $K_{n,m}$, together with eigenvectors.
2. Find the Laplacian eigenvalues of the complete graph K_n and of the complete bipartite graph $K_{n,m}$, together with eigenvectors.
3. Find the adjacency eigenvalues of the path graph P_n. Hint: use the spectrum of circulant matrices.
4. Find the Laplacian eigenvalues of the path graph P_n.
5. Find the adjacency eigenvalues of the $n \times n$ grid.

6. Find the modularity eigenvalues of the $n \times n$ grid.
7. Find the adjacency eigenvalues of the n-dimensional cube.
8. Find the modularity eigenvalues of the n-dimensional cube.
9. Find at least three cospectral trees on the same number of vertices (so that they have the same adjacency eigenvalues).
10. Prove the following: if a simple graph G on n vertices has an independent set of size $1 < k < n$, then $\lambda_{k-1} \leq 1$, where $0 = \lambda_0 \leq \lambda_1 \leq \cdots \leq \lambda_{n-1} \leq 2$ are the eigenvalues of the normalized Laplacian of G.