THE GENERALISED PRODUCT MOMENT DISTRIBUTION IN SAMPLES FROM A NORMAL MULTIVARIATE POPULATION.

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1. *Introduction.*

For some years prior to 1915, various writers struggled with the problems that arise when samples are taken from uni-variate and bi-variate populations, assumed in most cases for simplicity to be normal. Thus "Student," in 1908*, by considering the first four moments, was led by K. Pearson's methods to infer the distribution of standard deviations, in samples from a normal population. His results, for comparison with others to be deduced later, will be stated in the form

$$dp = \frac{1}{\Gamma\left(\frac{N-1}{2}\right)} A^{\frac{N-1}{2}} . e^{-Aa} . a^{\frac{N-3}{2}} da \qquad (1),$$

where N is the size of the sample, and

$$A = \frac{N}{2\sigma^2}, \quad a = s^2,$$

σ being the standard deviation of the sampled population, and s that estimated from the sample. Thus, if $x_1, x_2, \ldots x_N$ are the sample values,

$$N\bar{x} = \sum_1^N (x),$$

and

$$Ns^2 = \sum_1^N (x - \bar{x})^2.$$

When bi-variate populations were considered, other problems arose, such as the distribution of the correlation coefficient and of the regression coefficient in samples. These problems, taken by themselves, were found to be difficult, and only approximative results had been reached, when, in 1915, R. A. Fisher† gave a formula for the simultaneous distribution of the three quadratic statistical derivatives, namely the two variances (squared standard deviations) and the product moment coefficient. Thus, let $x_1, x_2, \ldots x_N$ represent the sample values of the x-variate, and $y_1, y_2, \ldots y$ the corresponding values for the y-variate, let σ_1 and σ_2 be the standard deviations of the sampled population and ρ the correlation between x and y. We then calculate the following statistical derivatives from the sample:

$$N\bar{x} = \sum_1^N (x) \qquad N\bar{y} = \sum_1^N (y)$$

$$Ns_1^2 = \sum_1^N (x - \bar{x})^2 \qquad Ns_2^2 = \sum_1^N (y - \bar{y})^2$$

$$Nrs_1s_2 = \sum_1^N (x - \bar{x})(y - \bar{y}).$$

* *Biometrika*, Vol. vi. 1908, pp. 4—6.

† *Biometrika*, Vol. x. 1915, p. 510.

If we put $A=\frac{N}{2\sigma_1^2(1-\rho^2)}$, $B=\frac{N}{2\sigma_2^2(1-\rho^2)}$, $H=-\frac{N\rho}{2\sigma_1\sigma_2(1-\rho^2)}$,

$a=s_1^2$, $b=s_2^2$, $h=s_1s_2r$,

then Fisher's result, for the simultaneous distribution of a, b and h, may be put in the symmetrical form*

$$dp=\frac{1}{\sqrt{\pi}\,\Gamma\left(\frac{N-1}{2}\right)\Gamma\left(\frac{N-2}{2}\right)}\begin{vmatrix} A & H \\ H & B\end{vmatrix}^{\frac{N-1}{2}}.e^{-Aa-Bb-2Hh}.\begin{vmatrix} a & h \\ h & b\end{vmatrix}^{\frac{N-4}{2}} dadbdh \quad ....(2).$$

The distribution of the correlation coefficient was deduced by direct integration from this result. Further, K. Pearson and V. Romanovsky, starting from this fundamental formula, were able to deal with the regression coefficients. Pearson, in 1925 †, gave the mean value and standard deviation of the regression coefficient, while Romanovsky and Pearson‡, in the following year, published the actual distribution.

New problems arise when we are dealing with three or more variates. There are, for example, the distributions of the partial and multiple correlations, and of the partial regression coefficients. In this domain our knowledge is still far from complete. The partial correlation coefficient has been shown to be distributed exactly as a total coefficient, when the size of the sample has been reduced by the number of variates eliminated§. The actual distribution of the multiple correlation coefficient, for the particular case where no real correlation exists in the sampled population, was shown in 1923‖ to be of the form

$$dp=\frac{\Gamma\left(\frac{N-1}{2}\right)}{\Gamma\left(\frac{N-n}{2}\right)\Gamma\left(\frac{n-1}{2}\right)}(R^2)^{\frac{n-3}{2}}(1-R^2)^{\frac{N-n-2}{2}}d(R^2) \quad ....(3),$$

where n is the total number of variates. A recent experimental investigation by the present writer¶ may be regarded as confirming this result, as well as was to be expected on the basis of the inadequate numbers used. On the other hand, L. Isserlis**, in 1917, found approximations to the mean value and standard deviation of the multiple correlation coefficient, in the case of three variates, while recently P. Hall††, for an n-fold system, determined (a) the mean value, and (b) the standard deviation, of the multiple and partial correlation coefficients, as far as terms of the order of $\frac{1}{N}$.

* See Romanovsky, *Comptes Rendus*, Tome 180, 1925, p. 1897.

† *Biometrika*, Vol. XVII. 1925, pp. 195—196.

‡ *Bulletin de l'Académie des Sciences de l'U.R.S.S.*, 1926, p. 646. *Proc. Roy. Soc.* A. 112, 1926, p. 1.

§ R. A. Fisher, *Metron*, Vol. III. Nos. 3—4, 1924, pp. 1—2.

‖ R. A. Fisher, *Phil. Trans.* B. Vol. 213, 1923, p. 91.

¶ *Memoirs of the Royal Meteorological Society*, Vol. II. pp. 29—37, 1928. *Rothamsted Memoirs*, Vol. XIV.

** *Phil. Mag.* Vol. XXXIV. 1917, pp. 205—220.

†† *Biometrika*, Vol. XIX. 1927, pp. 100—109.

What is now asserted is that all such problems depend, in the first instance, on the determination of a fundamental frequency distribution, which will be a generalisation of equation (2). It will, in fact, be the simultaneous distribution in samples of the n variances (squared standard deviations) and the $\frac{n(n-1)}{2}$ product moment coefficients. It is the purpose of the present paper to give this generalised distribution, and to calculate its moments up to the fourth order. The case of three variates will first be considered in detail, and thereafter a proof for the general n-fold system will be given.

2. *Tri-variate Product Moment Distribution.*

Let the frequency distribution of the population sampled be

$$dp = \frac{1}{(2\pi)^{\frac{3}{2}}\sigma_1\sigma_2\sigma_3\sqrt{\Delta}} \times e^{-\frac{1}{2\Delta}\left[\frac{(x-m_1)^2}{\sigma_1^2}\Delta_{11}+\frac{(y-m_2)^2}{\sigma_2^2}\Delta_{22}+\frac{(z-m_3)^2}{\sigma_3^2}\Delta_{33}+2\frac{(y-m_2)(z-m_3)}{\sigma_2\sigma_3}\Delta_{23}+2\frac{(z-m_3)(x-m_1)}{\sigma_3\sigma_1}\Delta_{31}+2\frac{(x-m_1)(y-m_2)}{\sigma_1\sigma_2}\Delta_{12}\right]} dx\,dy\,dz \quad \text{.........(4)},$$

where Δ is the determinant $|\rho_{st}|$ $s, t = 1, 2, 3$, and Δ_{st} is the minor of ρ_{st} in Δ.

Now let $x_1, x_2, \ldots x_N$ represent the sample values of the x-variate, and

$$y_1, y_2, \ldots y_N, \quad z_1, z_2, \ldots z_N$$

be the corresponding values for the y- and z-variates. Then the chance that

$$x_1, y_1, z_1, x_2, y_2, z_2, \ldots x_N, y_N, z_N$$

should fall within the elementary ranges $dx_1, dy_1, dz_1, \ldots dx_N, dy_N, dz_N$ is

$$dp = \frac{1}{(2\pi)^{\frac{3N}{2}}(\sigma_1\sigma_2\sigma_3)^N\Delta^{\frac{N}{2}}} \times e^{-\frac{1}{2\Delta}\sum_1^N\left[\frac{(x-m_1)^2}{\sigma_1^2}\Delta_{11}+\frac{(y-m_2)^2}{\sigma_2^2}\Delta_{22}+\frac{(z-m_3)^2}{\sigma_3^2}\Delta_{33}+2\frac{(y-m_2)(z-m_3)}{\sigma_2\sigma_3}\Delta_{23}+2\frac{(z-m_3)(x-m_1)}{\sigma_3\sigma_1}\Delta_{31}+2\frac{(x-m_1)(y-m_2)}{\sigma_1\sigma_2}\Delta_{12}\right]} dx_1dy_1dz_1dx_2dy_2dz_2 \ldots\ldots dx_Ndy_Ndz_N \quad \text{............(5)}.$$

The following statistical derivatives are now to be calculated from the sample and substituted in (5):

$$N\bar{x} = \sum_1^N(x) \qquad N\bar{y} = \sum_1^N(y) \qquad N\bar{z} = \sum_1^N(z)$$

$$Ns_1^2 = \sum_1^N(x-\bar{x})^2, \qquad Ns_2^2 = \sum_1^N(y-\bar{y})^2 \qquad Ns_3^2 = \sum_1^N(z-\bar{z})^2$$

$$Nr_{23}s_2s_3 = \sum_1^N(y-\bar{y})(z-\bar{z}) \quad Nr_{31}s_3s_1 = \sum_1^N(z-\bar{z})(x-\bar{x}) \quad Nr_{12}s_1s_2 = \sum_1^N(x-\bar{x})(y-\bar{y}).$$

In order to transform the element of volume, we require to extend somewhat the geometrical reasoning employed in the two variate case*. The N values of x may be regarded geometrically as specifying a point P in an N-dimensional space,

* See *Biometrika*, Vol. x. pp. 509—510.

whose co-ordinates are $x_1 - \bar{x}, x_2 - \bar{x}, \ldots x_N - \bar{x}$. Similarly the N values of y and the N values of z specify points Q, R in the same space. When $\bar{x}$ and s_1 are fixed, as when a particular sample is chosen, P is constrained to move so that its perpendicular distances from the line $x_1 = x_2 = \ldots = x_N$ and from the "plane"

$$x_1 + x_2 + \ldots + x_N = n\bar{x}$$

remain constant. It must therefore lie on the surface of an $N-1$-dimensional sphere which is everywhere at right angles to the radius vector $x_1 = x_2 = \ldots = x_N$. The element of volume is then proportional to $(\sqrt{N}s_1)^{N-2}\,ds_1 d\bar{x}$. For the factor of proportionality we require the entire area of the surface of a sphere in $N-1$ dimensions. If of radius r this is, for $N = 3, 4, 5, 6, 7$,

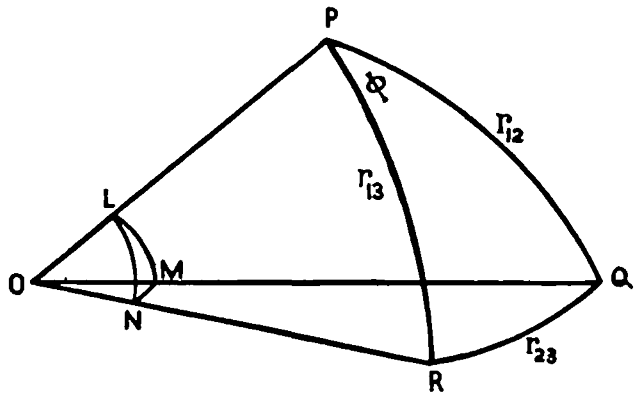
$$2\pi r,\quad 4\pi r^2,\quad 2\pi^2 r^3,\quad \tfrac{8}{3}\pi^2 r^4,\quad \pi^3 r^5,$$

the general result being
$$2 \cdot \frac{\pi^{\frac{N-1}{2}}}{\Gamma\left(\frac{N-1}{2}\right)} \cdot r^{N-2}.$$

Thus we have a contribution to the transformed element of volume of

$$2 \cdot \frac{\pi^{\frac{N-1}{2}} N^{\frac{N-2}{2}}}{\Gamma\left(\frac{N-1}{2}\right)} \cdot s_1^{N-2}\,ds_1\,d\bar{x}.$$

By similar reasoning Q and R must lie on concentric spheres in the same space, and there will be corresponding contributions to the transformed element of volume. Let the radii vectores OP, OQ, OR be cut by the unit sphere whose



centre is at O in the points L, M, N. Then LMN is a spherical triangle, specified by the nature of the sample. To find the chance that this particular triangle should be chosen we note that, P being fixed, the chance that Q (or M) should fall within the elementary range $d\theta_1$ (θ_1 being the angle $\frac{\pi}{2} - L\hat{O}M$) is equal to

$$\cos^{N-3}\theta_1 d\theta_1 \div \int_0^{\pi} \cos^{N-3}\theta_1 d\theta_1,$$

i.e. to
$$\frac{\Gamma\left(\frac{N-1}{2}\right)}{\sqrt{\pi}\,\Gamma\left(\frac{N-2}{2}\right)}\cos^{N-3}\theta_1\,d\theta_1.$$

Similarly the chance that R (or N) should fall within $d\theta_2\left(L\hat{O}N=\frac{\pi}{2}-\theta_2\right)$ is equal to
$$\frac{\Gamma\left(\frac{N-1}{2}\right)}{\sqrt{\pi}\,\Gamma\left(\frac{N-2}{2}\right)}\cos^{N-3}\theta_2\,d\theta_2.$$

But the points M and N do not vary independently. They are connected by the relation that, if ϕ be the angle between the planes LOM and LON,
$$\cos\phi=r_{23\cdot1}=\frac{r_{23}-r_{12}r_{13}}{\sqrt{1-r_{12}^2}\sqrt{1-r_{13}^2}}.$$

Now LM being fixed, the chance that LN should fall between the angles ϕ and $\phi+d\phi$, measured from LM, is equal to
$$\sin^{N-4}\phi\,d\phi\div\int_0^{\pi}\sin^{N-4}\phi\,d\phi=\frac{\Gamma\left(\frac{N-2}{2}\right)}{\sqrt{\pi}\,\Gamma\left(\frac{N-3}{2}\right)}\sin^{N-4}\phi\,d\phi.$$

The transformed volume element will consist of the product of all the above probabilities. The exponential term in (5) is easily expressed in terms of s_1, s_2, s_3 and the r's, and we have
$$dp=\frac{8}{\pi^3\,\Gamma\left(\frac{N-1}{2}\right)\Gamma\left(\frac{N-2}{2}\right)\Gamma\left(\frac{N-3}{2}\right)}\cdot\frac{N^{\frac{3(N-2)}{2}}}{(8\sigma_1^2\sigma_2^2\sigma_3^2\Delta)^{\frac{N}{2}}}$$
$$\times e^{-\frac{N}{2\Delta}\left[\frac{s_1^2+(\bar{x}-m_1)^2}{\sigma_1^2}\Delta_{11}+\frac{s_2^2+(\bar{y}-m_2)^2}{\sigma_2^2}\Delta_{22}+\frac{s_3^2+(\bar{z}-m_3)^2}{\sigma_3^2}\Delta_{33}+2\frac{r_{23}s_2s_3+(\bar{y}-m_2)(\bar{z}-m_3)}{\sigma_2\sigma_3}\Delta_{23}+2\frac{r_{31}s_3s_1+(\bar{z}-m_3)(\bar{x}-m_1)}{\sigma_3\sigma_1}\Delta_{31}+2\frac{r_{12}s_1s_2+(\bar{x}-m_1)(\bar{y}-m_2)}{\sigma_1\sigma_2}\Delta_{12}\right]}$$
$$\times s_1^{N-2}s_2^{N-2}s_3^{N-2}\cos^{N-3}\theta_1\cos^{N-3}\theta_2\sin^{N-4}\phi\,d\bar{x}\,d\bar{y}\,d\bar{z}\,ds_1\,ds_2\,ds_3\,d\theta_1\,d\theta_2\,d\phi$$
............(6).

We can integrate out immediately for $\bar{x}$, $\bar{y}$ and $\bar{z}$. The integration produces $(\sqrt{2\pi})^3N^{\frac{3}{2}}\sigma_1\sigma_2\sigma_3\sqrt{\Delta}$, and we get
$$dp=\frac{8}{\pi^3\,\Gamma\left(\frac{N-1}{2}\right)\Gamma\left(\frac{N-2}{2}\right)\Gamma\left(\frac{N-3}{2}\right)}\left(\frac{N^3}{8\sigma_1^2\sigma_2^2\sigma_3^2\Delta}\right)^{\frac{N-1}{2}}$$
$$\times e^{-\frac{N}{2\Delta}\left[\frac{s_1^2}{\sigma_1^2}\Delta_{11}+\frac{s_2^2}{\sigma_2^2}\Delta_{22}+\frac{s_3^2}{\sigma_3^2}\Delta_{33}+2\frac{r_{23}s_2s_3}{\sigma_2\sigma_3}\Delta_{23}+2\frac{r_{31}s_3s_1}{\sigma_3\sigma_1}\Delta_{31}+2\frac{r_{12}s_1s_2}{\sigma_1\sigma_2}\Delta_{12}\right]}$$
$$\times s_1^{N-2}s_2^{N-2}s_3^{N-2}\cos^{N-3}\theta_1\cos^{N-3}\theta_2\sin^{N-4}\phi\,ds_1\,ds_2\,ds_3\,d\theta_1\,d\theta_2\,d\phi$$
............(7).

Following Romanovsky* in the method of expression, let us put

$$\begin{aligned} s_1^2 &= a \qquad & s_1 s_2 \sin\theta_1 &= h, \\ s_2^2 &= b \qquad & s_1 s_3 \sin\theta_2 &= g, \\ s_3^2 &= c \qquad & s_2 s_3 (\cos\theta_1 \cos\theta_2 \cos\phi + \sin\theta_1 \sin\theta_2) &= f; \end{aligned}$$

$$\left(\text{for } r_{12} = \sin\theta_1,\ \ r_{13} = \sin\theta_2,\ \text{ and } \ r_{23\cdot1} = \cos\phi = \frac{r_{23} - r_{12} r_{13}}{\sqrt{1 - r_{12}^2}\sqrt{1 - r_{13}^2}}\right).$$

For the Jacobian of this final transformation we have

$$\frac{\partial(a, b, c, f, g, h)}{\partial(s_1, s_2, s_3, \theta_1, \theta_2, \phi)} = 8 s_1 s_2 s_3 \begin{vmatrix} \frac{\partial f}{\partial\theta_1} & 0 & \frac{\partial h}{\partial\theta_1} \\ \frac{\partial f}{\partial\theta_2} & \frac{\partial g}{\partial\theta_2} & 0 \\ \frac{\partial f}{\partial\phi} & 0 & 0 \end{vmatrix}$$

$$= 8 s_1^3 s_2^3 s_3^3 \cos^2\theta_1 \cos^2\theta_2 \sin\phi$$

$$= 8\,\{bc\,(ab - h^2)(ac - g^2)\,|\,abc\,|\}^{\frac{1}{2}},$$

where $|abc|$ denotes the determinant

$$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}.$$

The element of volume then becomes on transformation

$$\tfrac{1}{8}(abc)^{\frac{N-2}{2}} \frac{\{(ab - h^2)(ac - g^2)\}^{\frac{N-3}{2}}}{a^{N-3}(bc)^{\frac{N-3}{2}}} \cdot \frac{a^{\frac{N-4}{2}}\,|\,abc\,|^{\frac{N-4}{2}}}{\{(ab - h^2)(ac - g^2)\}^{\frac{N-4}{2}}} \cdot \frac{|\,abc\,|^{-\frac{1}{2}}}{(bc)^{\frac{1}{2}}\{(ab - h^2)(ac - g^2)\}^{\frac{1}{2}}}$$

$$= \tfrac{1}{8}\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}^{\frac{N-5}{2}},$$

a very neat result.

Finally we have for the simultaneous distribution of the three variances and the three product moment coefficients the symmetrical expression

$$dp = \frac{1}{\pi^{\frac{3}{2}}\,\Gamma\left(\frac{N-1}{2}\right)\Gamma\left(\frac{N-2}{2}\right)\Gamma\left(\frac{N-3}{2}\right)}\left(\frac{N^3}{8\sigma_1^2\sigma_2^2\sigma_3^2\Delta}\right)^{\frac{N-1}{2}}$$

$$\times\, e^{-\frac{N}{2\Delta}\left\{\frac{a}{\sigma_1^2}\Delta_{11} + \frac{b}{\sigma_2^2}\Delta_{22} + \frac{c}{\sigma_3^2}\Delta_{33} + 2\frac{f}{\sigma_2\sigma_3}\Delta_{23} + 2\frac{g}{\sigma_3\sigma_1}\Delta_{31} + 2\frac{h}{\sigma_1\sigma_2}\Delta_{12}\right\}}$$

$$\times \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}^{\frac{N-5}{2}} da\,db\,dc\,df\,dg\,dh.$$

* *Comptes Rendus*, loc. cit. See also *Metron*, Vol. v. No. 4, 1925, p. 31.

We may simplify this expression by writing

$$A=\frac{N}{2\sigma_1^2}\cdot\frac{\Delta_{11}}{\Delta},\quad B=\frac{N}{2\sigma_2^2}\cdot\frac{\Delta_{22}}{\Delta},\quad C=\frac{N}{2\sigma_3^2}\cdot\frac{\Delta_{33}}{\Delta},$$

$$F=\frac{N}{2\sigma_2\sigma_3}\cdot\frac{\Delta_{23}}{\Delta},\quad G=\frac{N}{2\sigma_3\sigma_1}\cdot\frac{\Delta_{31}}{\Delta},\quad H=\frac{N}{2\sigma_1\sigma_2}\cdot\frac{\Delta_{12}}{\Delta},$$

when it becomes

$$dp=\frac{1}{\pi^{\frac{3}{2}}\Gamma\left(\frac{N-1}{2}\right)\Gamma\left(\frac{N-2}{2}\right)\Gamma\left(\frac{N-3}{2}\right)}\begin{vmatrix}A & H & G\\ H & B & F\\ G & F & C\end{vmatrix}^{\frac{N-1}{2}}.e^{-Aa-Bb-Cc-2Ff-2Gg-2Hh}$$

$$\times\begin{vmatrix}a & h & g\\ h & b & f\\ g & f & c\end{vmatrix}^{\frac{N-5}{2}}dadbdcdfdgdh \quad \ldots\ldots\ldots\ldots(8).$$

It is to be noted that $|abc|$ is equal to $s_1^2s_2^2s_3^2\,|r_{pq}|$, $p, q=1, 2, 3$.

This is the fundamental frequency distribution for the three variate case, and in a later section the calculation of its moment coefficients will be dealt with.

3. *Multi-variate Distribution. Use of Quadratic co-ordinates.*

A comparison of equation (8) with the corresponding results (1) and (2) for uni-variate and bi-variate sampling, respectively, indicates the form the general result may be expected to take. In fact, we have for the simultaneous distribution in random samples of the n variances (squared standard deviations) and the $\frac{n(n-1)}{2}$ product moment coefficients the following expression:

$$dp=\frac{\begin{vmatrix}A_{11} & A_{12}\ldots A_{1n}\\ A_{21} & A_{22}\ldots A_{2n}\\ \vdots & \vdots \quad\quad \vdots\\ A_{n1} & A_{n2}\ldots A_{nn}\end{vmatrix}^{\frac{N-1}{2}}}{(\sqrt{\pi})^{\frac{1}{2}n(n-1)}\Gamma\left(\frac{N-1}{2}\right)\Gamma\left(\frac{N-2}{2}\right)\ldots\Gamma\left(\frac{N-n}{2}\right)}$$

$$\times e^{-A_{11}a_{11}-A_{22}a_{22}-\ldots-A_{nn}a_{nn}-2A_{12}a_{12}-2A_{13}a_{13}-\ldots-2A_{n-1n}a_{n-1n}}$$

$$\times\begin{vmatrix}a_{11} & a_{12}\ldots a_{1n}\\ a_{21} & a_{22}\ldots a_{2n}\\ \vdots & \vdots \quad\quad \vdots\\ a_{n1} & a_{n2}\ldots a_{nn}\end{vmatrix}^{\frac{N-n-2}{2}}da_{11}da_{12}\ldots da_{nn}\quad\ldots\ldots\ldots\ldots\ldots\ldots(9),$$

where $a_{pq}=s_ps_qr_{pq}$, and $A_{pq}=\frac{N}{2\sigma_p\sigma_q}\cdot\frac{\Delta_{pq}}{\Delta}$, Δ being the determinant

$$|\rho_{pq}|,\ p, q=1, 2, 3, \ldots n,$$

and Δ_{pq} the minor of ρ_{pq} in Δ.

This result can be proved by the aid of the following general geometrical considerations. We shall begin by defining the *quadratic co-ordinates* of a set of points, and thereafter develop the argument by the use of these co-ordinates.

(*a*) If $x_{p1}, x_{p2}, \ldots x_{pn}$ are the rectangular co-ordinates of a point (p) in space of n dimensions, then the configuration of a set of n points relative to the origin may be specified by the co-ordinates

$$\xi_{pq} = \sum_{k=1}^{n} (x_{pk} x_{qk}).$$

These co-ordinates will be unchanged by any rotation of the whole system about the origin, the configuration of the set of $(n+1)$ points including the origin being unchanged.

(*b*) The determinant $|\xi_{pq}|$, $p, q = 1, 2, 3, \ldots n$, on expanding the several terms, may be recognised as the square of the determinant $|x_{pq}|$, $p, q = 1, 2, 3, \ldots n$, and is therefore equal to the square of the volume enclosed by completing the parallel-faced figure having one corner at the origin and edges running to the n points of the figure. This volume will be represented by v_n.

(*c*) The perpendicular distance of the point (n) from the plane space passing through the origin and the points (1) to $(n-1)$ is $\frac{v_n}{v_{n-1}}$.

(*d*) If the points (1) to $(n-1)$ are fixed, and that of the point (n) is specified by the co-ordinates $\xi_{n1}, \xi_{n2}, \ldots \xi_{nn}$, then the element of volume in the neighbourhood of the point (n) may be represented by

$$\frac{1}{2v_n} d\xi_{n1}, d\xi_{n2} \ldots d\xi_{nn}.$$

For the Jacobian

$$\frac{\partial(\xi_{n1}, \xi_{n2}, \ldots \xi_{nn})}{\partial(x_{n1}, x_{n2}, \ldots x_{nn})} = \begin{vmatrix} x_{11} & x_{12} \ldots & x_{1n} \\ x_{12} & x_{22} \ldots & x_{2n} \\ \vdots & \vdots & \vdots \\ 2x_{1n} & 2x_{2n} & 2x_{nn} \end{vmatrix} = 2v_n.$$

(*e*) If in a space of N dimensions the points (1) to $(n-1)$ are fixed, and that of the point (n) is specified by the co-ordinates $\xi_{n1}, \xi_{n2}, \ldots \xi_{nn}$ within ranges

$$d\xi_{n1}, d\xi_{n2}, \ldots d\xi_{nn},$$

then the point (n) is free to move on the surface of a sphere in $N-n+1$ dimensions, of which the centre is the foot of the perpendicular from (n) upon the space containing the points (1) to $(n-1)$. The radius of this sphere is therefore $\frac{v_n}{v_{n-1}}$, and the area of its surface is

$$2 \cdot \frac{\pi^{\frac{1}{2}(N-n+1)}}{\Gamma\left(\frac{N-n+1}{2}\right)} \left(\frac{v_n}{v_{n-1}}\right)^{N-n} \quad \text{(see above, p. 35).}$$

This surface is everywhere normal to the space passing through the origin and the points (1) to (n). But the element of volume in this space is

$$\frac{1}{2v_n}\, d\xi_{n1},\, d\xi_{n2},\, \ldots\, d\xi_{nn}.$$

Hence the entire volume that may be occupied by (n) is

$$\frac{\pi^{\frac{1}{2}(N-n+1)}}{\Gamma\left(\frac{N-n+1}{2}\right)} \cdot \frac{v_n^{\,N-n-1}}{v_{n-1}^{\,N-n}} \cdot d\xi_{n1}, d\xi_{n2}, \ldots\, d\xi_{nn} \quad \ldots\ldots\ldots\ldots(10).$$

(f) The quadratic co-ordinates of n points in space of N dimensions may be also regarded as co-ordinates of a single point in space of n sets, each of N dimensions. If the projections of the point upon the first $n-1$ spaces are fixed, the volume element corresponding to variations of the co-ordinates ξ_{n1}, ξ_{n2}, ... ξ_{nn} will be that found above.

(g) We now require the volume element corresponding to variations of all the co-ordinates of a point so specified in Nn dimensions. The component spaces of N dimensions each are defined by rectangular co-ordinates, hence the entire volume may be found by multiplying the volume element defined by the variations of ξ_{11}, ξ_{12}, ... $\xi_{n-1\,n-1}$ by that defined by the variations of ξ_{n1}, ξ_{n2},' ... ξ_{nn}. Consequently the volume element for variations of all the co-ordinates is found by multiplying together the volumes (equation (10)) got by putting n successively equal to 1, 2, ... n. The component volumes v_{n-r} appear in these expressions successively in the numerator and the denominator with equal indices, and therefore disappear from the product, thus we have finally for the volume element

$$\frac{\pi^{\frac{1}{4}n(2N-n+1)}}{\Gamma\left(\frac{N}{2}\right)\Gamma\left(\frac{N-1}{2}\right)\ldots\Gamma\left(\frac{N-n+1}{2}\right)}\, v_n^{\,N-n-1}\, d\xi_{11}, d\xi_{12}, \ldots\, d\xi_{nn} \quad \ldots(11).$$

(h) Now write $N-1$ for N and let $\xi_{pq} = Na_{pq}$, to correspond with our earlier work. The above expression then becomes

$$\frac{\pi^{\frac{1}{4}n(2N-n-1)} \cdot N^{\frac{1}{2}n(N-1)}}{\Gamma\left(\frac{N-1}{2}\right)\Gamma\left(\frac{N-2}{2}\right)\ldots\Gamma\left(\frac{N-n}{2}\right)} \begin{vmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \ldots & a_{nn} \end{vmatrix}^{\frac{N-n-2}{2}} da_{11} da_{12} \ldots\, da_{nn},$$

and this multiplied by a density factor

$$(2\pi)^{-\frac{n}{2}(N-1)} \cdot \left(\frac{2}{N}\right)^{\frac{n}{2}(N-1)}$$

becomes

$$\frac{1}{(\sqrt{\pi})^{\frac{n(n-1)}{2}}\,\Gamma\left(\frac{N-1}{2}\right)\Gamma\left(\frac{N-2}{2}\right)\ldots\Gamma\left(\frac{N-n}{2}\right)} \begin{vmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \ldots & a_{nn} \end{vmatrix}^{\frac{N-n-2}{2}} da_{11} da_{12} \ldots\, da_{nn} \quad \ldots\ldots\ldots\ldots(12),$$

whence our general result (9) follows.

4. *Moment Coefficients of the Distribution.*

It is now possible to find the moments, up to any desired order, of the simultaneous distribution given by (9). A number of these moments will be identical with those found previously for the cases of one and two variates, and published by different writers. To illustrate for the case of three variates, let

$$F(abcfgh) = \frac{1}{\pi^{\frac{3}{2}}\Gamma\left(\frac{N-1}{2}\right)\Gamma\left(\frac{N-2}{2}\right)\Gamma\left(\frac{N-3}{2}\right)}\begin{vmatrix} A & H & G \\ H & B & F \\ G & F & C \end{vmatrix}^{\frac{N-1}{2}} \times e^{-Aa-Bb-Cc-2Ff-2Gg-2Hh}\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}^{\frac{N-5}{2}} \quad \ldots\ldots\ldots\ldots(13).$$

Then the generalised product moment coefficient of this distribution is given by

$$M_{jklmpq} = \iiiiiint a^j b^k c^l f^m g^p h^q \, F(abcfgh)\, da\, db\, dc\, df\, dg\, dh,$$

where the limits of integration for a, b and c are 0 to ∞, and for f, g and h $-\sqrt{bc}$ to $+\sqrt{bc}$, $-\sqrt{ac}$ to $+\sqrt{ac}$ and $-\sqrt{ab}$ to $+\sqrt{ab}$ respectively. Now following the method of Romanovsky*, we define the function

$$\phi(\alpha\beta\gamma\lambda\mu\nu) = \int_0^\infty da \int_0^\infty db \int_0^\infty dc \iiint F(abcfgh)\, e^{a\alpha+b\beta+c\gamma+f\lambda+g\mu+h\nu}\, df\, dg\, dh \quad \ldots\ldots\ldots\ldots(14)$$

as the generating function of the product moments, which are given by

$$M_{jklmpq} = \left[\frac{\partial^{j+k+l+m+p+q}\,\phi(\alpha\beta\gamma\lambda\mu\nu)}{\partial\alpha^j\partial\beta^k\partial\gamma^l\partial\lambda^m\partial\mu^p\partial\nu^q}\right]_{\alpha=\beta=\gamma=\lambda=\mu=\nu=0} \quad \ldots\ldots\ldots(15).$$

Since the total frequency volume in (8) is unity, we have

$$\int e^{-Aa-Bb-Cc-2Ff-2Gg-2Hh}\, dv = \text{const.}\begin{vmatrix} A & H & G \\ H & B & F \\ G & F & C \end{vmatrix}^{-\frac{N-1}{2}}$$

where the integration is taken throughout the volume. A parallel result is obtained for the integration in (14) simply by replacing A by $A-\alpha$, B by $B-\beta$, H by $H-\frac{1}{2}\nu$, etc., in the determinant $|ABC|$. What is then left to deal with is the fundamental frequency volume and we have

$$\phi(\alpha\beta\gamma\lambda\mu\nu) = \begin{vmatrix} A & H & G \\ H & B & F \\ G & F & C \end{vmatrix}^{\frac{N-1}{2}} \times \begin{vmatrix} A-\alpha & H-\frac{1}{2}\nu & G-\frac{1}{2}\mu \\ H-\frac{1}{2}\nu & B-\beta & F-\frac{1}{2}\lambda \\ G-\frac{1}{2}\mu & F-\frac{1}{2}\lambda & C-\gamma \end{vmatrix}^{-\frac{N-1}{2}} \quad \ldots(16),$$

a convenient expression from which to calculate the moments. It is of the same form when generalised for the case of n variates.

We shall use the notation $\mu\begin{pmatrix} j & q & p \\ & k & m \\ & & l \end{pmatrix}$,

corresponding to the determinant $\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}$,

* *Metron*, loc. cit. pp. 27, 32, 35—6.

to represent the general product moment coefficient of the jth power of s_1^2, kth power of s_2^2, lth power of s_3^2, mth power of $s_2 s_3 r_{23}$, pth power of $s_1 s_3 r_{13}$ and qth power of $s_1 s_2 r_{12}$. These moment coefficients are in all cases except the first calculated about the mean of the sample, and are expressed in terms of the following moment coefficients of the general population:

$$\mu_{200} = \sigma_1^2, \qquad \mu_{020} = \sigma_2^2, \qquad \mu_{002} = \sigma_3^2,$$

$$\mu_{110} = \sigma_1\sigma_2\rho_{12}, \quad \mu_{101} = \sigma_1\sigma_3\rho_{13}, \quad \mu_{011} = \sigma_2\sigma_3\rho_{23}.$$

In most cases there will be a number of similar results deduced from one another merely by a cyclical interchange of the suffixes, but one representative result for each class only will be stated. The following pages list every independent result up to the fourth order, and as far as eight variates. They therefore embody certain results which are already known. "Student"* first gave the one-variate moments up to the fourth order, while Soper†, to his degree of approximation, gave all the two-variate results, to the same order. His second and third order moments agree with the following, except for the terms in N, which he makes $\frac{1}{N}$ and $\frac{1}{N^2}$, in place of $\frac{N-1}{N^2}$ and $\frac{N-1}{N^3}$. The divergence of the fourth order results is wider, for the same reason. Isserlis‡ later gave the general 2nd order—4 variate, 3rd order—6 variate and 4th order—8 variate moments, sampling from a limited population. For an infinite normal population his results can be expressed in terms of the ρ's, and, of course, all the moments up to the fourth order can be deduced from his three results by associating the variates in all the possible ways. But his results were calculated about the general population value, and not, as in the following, about the mean of the sample.

5. *Derived Coefficients.*

We are now able to deduce certain other constants of the distribution curves specified by the moments cited. In particular we have for the Betas of a_{12}, i.e. of $s_1 s_2 r_{12}$,

$$_{a_{12}}\beta_1 = \left\{\mu\left(\begin{smallmatrix}\cdot & 3\\ & \cdot\end{smallmatrix}\right)\right\}^2 \div \left\{\mu\left(\begin{smallmatrix}\cdot & 2\\ & \cdot\end{smallmatrix}\right)\right\}^3 = \frac{4}{N-1}\cdot\frac{\rho_{12}^2(3+\rho_{12}^2)^2}{(1+\rho_{12}^2)^3},$$

$$_{a_{12}}\beta_2 = \left\{\mu\left(\begin{smallmatrix}\cdot & 4\\ & \cdot\end{smallmatrix}\right)\right\} \div \left\{\mu\left(\begin{smallmatrix}\cdot & 2\\ & \cdot\end{smallmatrix}\right)\right\}^2 = 3 + \frac{6}{N-1}\cdot\frac{1+6\rho_{12}^2+\rho_{12}^4}{(1+\rho_{12}^2)^2}.$$

These values show that when ρ_{12}, the correlation coefficient of the sampled population, is zero, the distribution of a_{12} is symmetrical, but not normal. For

$$\beta_2 = 3 + \frac{6}{N-1},$$

showing that as far as the first four moments the curve agrees with Type VII. On

* *Biometrika*, Vol. VI. 1908, p. 4.

† *Biometrika*, Vol. IX. 1913, pp. 103—104.

‡ *Biometrika*, Vol. XII. 1918, p. 188.

the other hand, when $\rho_{12}=1$ we have the Betas of s^2, the square of the standard deviation of the sample,

$$_{s^2}\beta_1=\frac{8}{N-1},\quad {}_{s^2}\beta_2=3+\frac{12}{N-1},$$

the well-known Type III result*.

Since $_{a_{12}}\beta_2$ is always greater than 3, the curve of distribution is leptokurtic. We also have

$$2\beta_2-3\beta_1-6=\frac{12}{N-1}\cdot\frac{(1-\rho^2)^2}{(1+\rho^2)^2},$$

which is always positive, having a maximum value $\frac{12}{N-1}$. Also

$$k_2=\frac{\rho^2(3+\rho^2)^2}{4(1-\rho^2)^2}\left\{1+\frac{1}{N-1}\cdot\frac{1+6\rho^2+\rho^4}{(1+\rho^2)^2}\right\}^2\div\left\{1+\frac{1}{N-1}\cdot\frac{2+5\rho^2+8\rho^4+\rho^6}{(1+\rho^2)^3}\right\},$$

showing that the curve is of Type IV for small ρ, but beyond a value of about ·45 (according to the size of the sample) it becomes Type VI.

Certain correlation coefficients also follow at once from the moments given. It will be remembered that Pearson and Filon in *Phil. Trans.* Vol. 191 A, 1898, p. 242, gave $r_{\sigma_1\sigma_2}=\rho_{12}^2$ and $r_{\sigma_1 r_{12}}=\rho_{12}/\sqrt{2}$ approximately. They also gave approximate expressions for $r_{\sigma_1 . r_{23}}$, $r_{r_{12}.r_{13}}$ and $r_{r_{12}.r_{34}}$ (pp. 256, 259, 262). In our case the correlations we are able to deduce are those between the product moments themselves and are exact, thus

$$r_{a_{11}.a_{22}}=\left\{\mu\begin{pmatrix}1&\\&1\end{pmatrix}\right\}\div\left\{\mu\begin{pmatrix}2&\\&\cdot\end{pmatrix}\right\}^{\frac{1}{2}}\left\{\mu\begin{pmatrix}\cdot&\\&2\end{pmatrix}\right\}^{\frac{1}{2}}=\rho_{12}^2,$$

$$r_{a_{11}.a_{12}}=\left\{\mu\begin{pmatrix}1&1\\&\cdot\end{pmatrix}\right\}\div\left\{\mu\begin{pmatrix}2&\cdot\\&\cdot\end{pmatrix}\right\}^{\frac{1}{2}}\left\{\mu\begin{pmatrix}\cdot&2\\&\cdot\end{pmatrix}\right\}^{\frac{1}{2}}=\left\{\frac{2\rho_{12}^2}{1+\rho_{12}^2}\right\}^{\frac{1}{2}},$$

$$r_{a_{11}.a_{23}}=\left\{\mu\begin{pmatrix}1&\cdot&\cdot\\&\cdot&1\\&&\cdot\end{pmatrix}\right\}\div\left\{\mu\begin{pmatrix}2&\cdot&\cdot\\&\cdot&\cdot\\&&\cdot\end{pmatrix}\right\}^{\frac{1}{2}}\left\{\mu\begin{pmatrix}\cdot&\cdot&\cdot\\&\cdot&2\\&&\cdot\end{pmatrix}\right\}^{\frac{1}{2}}=\left\{\frac{2\rho_{12}^2\rho_{13}^2}{1+\rho_{23}^2}\right\}^{\frac{1}{2}},$$

$$r_{a_{23}.a_{13}}=\left\{\mu\begin{pmatrix}\cdot&\cdot&1\\&\cdot&1\\&&\cdot\end{pmatrix}\right\}\div\left\{\mu\begin{pmatrix}\cdot&\cdot&2\\&\cdot&\cdot\\&&\cdot\end{pmatrix}\right\}^{\frac{1}{2}}\left\{\mu\begin{pmatrix}\cdot&\cdot&\cdot\\&\cdot&2\\&&\cdot\end{pmatrix}\right\}^{\frac{1}{2}}=\frac{\rho_{12}+\rho_{13}\rho_{23}}{\{(1+\rho_{23}^2)(1+\rho_{13}^2)\}^{\frac{1}{2}}},$$

$$r_{a_{12}.a_{34}}=\left\{\mu\begin{pmatrix}\cdot&1&\cdot&\cdot\\&\cdot&\cdot&\cdot\\&&\cdot&1\\&&&\cdot\end{pmatrix}\right\}\div\left\{\mu\begin{pmatrix}\cdot&2&\cdot&\cdot\\&\cdot&\cdot&\cdot\\&&\cdot&\cdot\\&&&\cdot\end{pmatrix}\right\}^{\frac{1}{2}}\left\{\mu\begin{pmatrix}\cdot&\cdot&\cdot&\cdot\\&\cdot&\cdot&\cdot\\&&\cdot&2\\&&&\cdot\end{pmatrix}\right\}^{\frac{1}{2}}=\frac{\rho_{13}\rho_{24}+\rho_{14}\rho_{23}}{\{(1+\rho_{12}^2)(1+\rho_{34}^2)\}^{\frac{1}{2}}}.$$

My thanks are due to Dr R. A. Fisher, in whose laboratory this paper was written, and without whose critical help it would have been difficult to generalise the geometrical methods employed by him.

* "Student," *Biometrika*, Vol. VI. 1908, p. 4.

1ST ORDER:

1 *variate*

(1) $\mu(1) = \frac{N-1}{N}\sigma_1^2$.

2 *variates*

(2) $\mu\begin{pmatrix}\cdot & 1\\ & \cdot\end{pmatrix} = \frac{N-1}{N}\sigma_1\sigma_2\rho_{12}$.

2ND ORDER:

1 *variate*

(3) $\mu(2) = 2\frac{N-1}{N^2}\sigma_1^4$.

2 *variates*

(4) $\mu\begin{pmatrix}1 & \\ & 1\end{pmatrix} = 2\frac{N-1}{N^2}\sigma_1^2\sigma_2^2\rho_{12}^2$,

(5) $\mu\begin{pmatrix}\cdot & 2\\ & \cdot\end{pmatrix} = \frac{N-1}{N^2}\sigma_1^2\sigma_2^2(1+\rho_{12}^2)$,

(6) $\mu\begin{pmatrix}1 & 1\\ & \cdot\end{pmatrix} = 2\frac{N-1}{N^2}\sigma_1^3\sigma_2\rho_{12}$.

3 *variates*

(7) $\mu\begin{pmatrix}1 & \cdot & \cdot\\ & \cdot & 1\\ & & \cdot\end{pmatrix} = 2\frac{N-1}{N^2}\sigma_1^2\sigma_2\sigma_3\rho_{12}\rho_{13}$,

(8) $\mu\begin{pmatrix}\cdot & \cdot & 1\\ & \cdot & 1\\ & & \cdot\end{pmatrix} = \frac{N-1}{N^2}\sigma_1\sigma_2\sigma_3^2(\rho_{12}+\rho_{13}\rho_{23})$.

4 *variates*

(9) $\mu\begin{pmatrix}\cdot & 1 & \cdot & \cdot\\ & \cdot & \cdot & \cdot\\ & & \cdot & 1\\ & & & \cdot\end{pmatrix} = \frac{N-1}{N^2}\sigma_1\sigma_2\sigma_3\sigma_4(\rho_{13}\rho_{24}+\rho_{14}\rho_{23})$.

3RD ORDER:

1 *variate*

(10) $\mu(3) = 8\frac{N-1}{N^3}\sigma_1^6$.

2 *variates*

(11) $\mu\begin{pmatrix}2 & \\ & 1\end{pmatrix} = 8\frac{N-1}{N^3}\sigma_1^4\sigma_2^2\rho_{12}^2$,

(12) $\mu\begin{pmatrix}\cdot & 3\\ & \cdot\end{pmatrix} = 2\frac{N-1}{N^3}\sigma_1^3\sigma_2^3\rho_{12}(3+\rho_{12}^2)$,

(13) $\mu\begin{pmatrix}1 & 2\\ & \cdot\end{pmatrix} = 2\frac{N-1}{N^3}\sigma_1^4\sigma_2^2(1+3\rho_{12}^2)$,

(14) $\mu\begin{pmatrix}1 & 1\\ & 1\end{pmatrix} = 4\frac{N-1}{N^3}\sigma_1^3\sigma_2^3\rho_{12}(1+\rho_{12}^2)$,

(15) $\mu\begin{pmatrix}2 & 1\\ & \cdot\end{pmatrix} = 8\frac{N-1}{N^3}\sigma_1^5\sigma_2\rho_{12}$.

3 *variates*

(16) $\mu\begin{pmatrix}2 & \cdot & \cdot\\ & \cdot & 1\\ & & \cdot\end{pmatrix} = 8\frac{N-1}{N^3}\sigma_1^4\sigma_2\sigma_3\rho_{12}\rho_{13}$,

(17) $\mu\begin{pmatrix}1 & \cdot & \cdot\\ & \cdot & 2\\ & & \cdot\end{pmatrix} = 2\frac{N-1}{N^3}\sigma_1^2\sigma_2^2\sigma_3^2(\rho_{12}^2+\rho_{13}^2+2\rho_{12}\rho_{13}\rho_{23})$,

(18) $\mu\begin{pmatrix}\cdot & \cdot & 1\\ & \cdot & 2\\ & & \cdot\end{pmatrix} = 2\frac{N-1}{N^3}\sigma_1\sigma_2^2\sigma_3^3(\rho_{13}+2\rho_{12}\rho_{23}+\rho_{13}\rho_{23}^2)$,

(19) $\mu\begin{pmatrix}1 & \cdot & \cdot\\ & 1 & \cdot\\ & & 1\end{pmatrix} = 8\frac{N-1}{N^3}\sigma_1^2\sigma_2^2\sigma_3^2\rho_{12}\rho_{13}\rho_{23}$,

(20) $\mu\begin{pmatrix}1 & 1 & 1\\ & \cdot & \cdot\\ & & \cdot\end{pmatrix} = 2\frac{N-1}{N^3}\sigma_1^4\sigma_2\sigma_3(\rho_{23}+3\rho_{12}\rho_{13})$,

(21) $\mu\begin{pmatrix}\cdot & 1 & 1\\ & \cdot & 1\\ & & \cdot\end{pmatrix} = \frac{N-1}{N^3}\sigma_1^2\sigma_2^2\sigma_3^2(1+\rho_{12}^2+\rho_{13}^2+\rho_{23}^2+4\rho_{12}\rho_{13}\rho_{23})$,

(22) $\mu\begin{pmatrix}1 & \cdot & \cdot\\ & 1 & 1\\ & & \cdot\end{pmatrix} = 4\frac{N-1}{N^3}\sigma_1^2\sigma_2^3\sigma_3\rho_{12}(\rho_{13}+\rho_{12}\rho_{23})$,

(23) $\mu\begin{pmatrix}1 & \cdot & 1\\ & \cdot & 1\\ & & \cdot\end{pmatrix} = 2\frac{N-1}{N^3}\sigma_1^3\sigma_2\sigma_3^2(\rho_{12}+\rho_{13}\rho_{23}+2\rho_{12}\rho_{13}^2)$.

4 variates

(24) $\mu\begin{pmatrix}1 & \cdot & \cdot & \cdot \\ & 1 & \cdot & \cdot \\ & & \cdot & 1 \\ & & & \cdot\end{pmatrix} = 4\,\dfrac{N-1}{N^3}\,\sigma_1^2\sigma_2^2\sigma_3\sigma_4\rho_{12}(\rho_{13}\rho_{24}+\rho_{14}\rho_{23}),$

(25) $\mu\begin{pmatrix}1 & \cdot & \cdot & \cdot \\ & \cdot & 1 & \cdot \\ & & \cdot & 1 \\ & & & \cdot\end{pmatrix} = 2\,\dfrac{N-1}{N^3}\,\sigma_1^2\sigma_2\sigma_3^2\sigma_4[\rho_{12}\rho_{14}+\rho_{13}(\rho_{12}\rho_{34}+\rho_{13}\rho_{24}+\rho_{14}\rho_{23})],$

(26) $\mu\begin{pmatrix}\cdot & 1 & 1 & 1 \\ & \cdot & \cdot & \cdot \\ & & \cdot & \cdot \\ & & & \cdot\end{pmatrix} = 2\,\dfrac{N-1}{N^3}\,\sigma_1^2\sigma_2\sigma_3\sigma_4(\rho_{12}\rho_{34}+\rho_{13}\rho_{24}+\rho_{14}\rho_{23}+\rho_{12}\rho_{13}\rho_{14}),$

(27) $\mu\begin{pmatrix}1 & 1 & \cdot & \cdot \\ & \cdot & \cdot & \cdot \\ & & \cdot & 1 \\ & & & \cdot\end{pmatrix} = 2\,\dfrac{N-1}{N^3}\,\sigma_1^3\sigma_2\sigma_3\sigma_4(\rho_{13}\rho_{24}+\rho_{14}\rho_{23}+2\rho_{12}\rho_{13}\rho_{14}),$

(28) $\mu\begin{pmatrix}\cdot & 2 & \cdot & \cdot \\ & \cdot & \cdot & \cdot \\ & & \cdot & 1 \\ & & & \cdot\end{pmatrix} = 2\,\dfrac{N-1}{N^3}\,\sigma_1^2\sigma_2^2\sigma_3\sigma_4[\rho_{13}\rho_{14}+\rho_{23}\rho_{24}+\rho_{12}(\rho_{13}\rho_{24}+\rho_{14}\rho_{23})],$

(29) $\mu\begin{pmatrix}\cdot & 1 & 1 & \cdot \\ & \cdot & \cdot & 1 \\ & & \cdot & \cdot \\ & & & \cdot\end{pmatrix} = \dfrac{N-1}{N^3}\,\sigma_1^2\sigma_2^2\sigma_3\sigma_4[\rho_{34}+\rho_{13}\rho_{14}+\rho_{23}\rho_{24}+\rho_{12}(\rho_{12}\rho_{34}+\rho_{13}\rho_{24}+3\rho_{14}\rho_{23})].$

5 variates

(30) $\mu\begin{pmatrix}1 & \cdot & \cdot & \cdot & \cdot \\ & \cdot & 1 & \cdot & \cdot \\ & & \cdot & \cdot & \cdot \\ & & & \cdot & 1 \\ & & & & \cdot\end{pmatrix} = 2\,\dfrac{N-1}{N^3}\,\sigma_1^2\sigma_2\sigma_3\sigma_4\sigma_5[\rho_{12}(\rho_{14}\rho_{35}+\rho_{15}\rho_{34})+\rho_{13}(\rho_{14}\rho_{25}+\rho_{15}\rho_{24})],$

(31) $\mu\begin{pmatrix}\cdot & 1 & \cdot & \cdot & \cdot \\ & \cdot & 1 & \cdot & \cdot \\ & & \cdot & \cdot & \cdot \\ & & & \cdot & 1 \\ & & & & \cdot\end{pmatrix} = \dfrac{N-1}{N^3}\,\sigma_1\sigma_2^2\sigma_3\sigma_4\sigma_5[\rho_{14}\rho_{35}+\rho_{15}\rho_{34}+2\rho_{13}\rho_{24}\rho_{25}+\rho_{12}(\rho_{24}\rho_{35}+\rho_{25}\rho_{34})$

$+\rho_{23}(\rho_{14}\rho_{25}+\rho_{15}\rho_{24})].$

6 variates

(32) $\mu\begin{pmatrix}\cdot & 1 & \cdot & \cdot & \cdot & \cdot \\ & \cdot & \cdot & \cdot & \cdot & \cdot \\ & & \cdot & 1 & \cdot & \cdot \\ & & & \cdot & \cdot & \cdot \\ & & & & \cdot & 1 \\ & & & & & \cdot\end{pmatrix} = \dfrac{N-1}{N^3}\,\sigma_1\sigma_2\sigma_3\sigma_4\sigma_5\sigma_6[\rho_{12}(\rho_{35}\rho_{46}+\rho_{36}\rho_{45})+\rho_{14}(\rho_{25}\rho_{36}+\rho_{26}\rho_{35})$

$+\rho_{15}(\rho_{23}\rho_{46}+\rho_{24}\rho_{36})+\rho_{16}(\rho_{23}\rho_{45}+\rho_{24}\rho_{35})].$

4TH ORDER:

1 variate

(33) $\mu(4) = 12\,\dfrac{(N-1)(N+3)}{N^4}\,\sigma_1^8.$

2 variates

(34) $\mu\begin{pmatrix}3 & \\ & 1\end{pmatrix} = 12\,\dfrac{(N-1)(N+3)}{N^4}\,\sigma_1^6\sigma_2^2\rho_{12}^2,$

(35) $\mu\begin{pmatrix}2 & \\ & 2\end{pmatrix} = 4\,\dfrac{N-1}{N^4}\,\sigma_1^4\sigma_2^4[N-1+8\rho_{12}^2+2(N+1)\rho_{12}^4],$

(36) $\mu\begin{pmatrix}2 & 1 \\ & 1\end{pmatrix} = 4\,\dfrac{(N-1)(N+3)}{N^4}\,\sigma_1^5\sigma_2^3\rho_{12}(1+2\rho_{12}^2),$

(37) $\mu\begin{pmatrix}1 & 3 \\ & \cdot\end{pmatrix} = 6\,\dfrac{(N-1)(N+3)}{N^4}\,\sigma_1^5\sigma_2^3\rho_{12}(1+\rho_{12}^2),$

(38) $\mu\begin{pmatrix}3 & 1 \\ & \cdot\end{pmatrix} = 12\,\dfrac{(N-1)(N+3)}{N^4}\,\sigma_1^7\sigma_2\rho_{12},$

(39) $\mu\begin{pmatrix}2 & 2 \\ & \cdot\end{pmatrix} = 2\,\dfrac{(N-1)(N+3)}{N^4}\,\sigma_1^6\sigma_2^2(1+5\rho_{12}^2),$

(40) $\mu\begin{pmatrix}1 & 2 \\ & 1\end{pmatrix} = 2\,\dfrac{N-1}{N^4}\,\sigma_1^4\sigma_2^4[2+(5N+11)\rho_{12}^2+(N+5)\rho_{12}^4],$

(41) $\mu\begin{pmatrix}\cdot & 4 \\ & \cdot\end{pmatrix} = 3\,\dfrac{N-1}{N^4}\,\sigma_1^4\sigma_2^4[(N+1)(1+\rho_{12}^4)+2(N+5)\rho_{12}^2].$

3 *variates*

(42) $\mu\begin{pmatrix} 3 & \cdot & \cdot \\ & \cdot & 1 \\ & & \cdot \end{pmatrix} = 12\,\frac{(N-1)(N+3)}{N^4}\,\sigma_1^6\sigma_2\sigma_3\rho_{12}\rho_{13}$,

(43) $\mu\begin{pmatrix} 2 & \cdot & \cdot \\ & 1 & \cdot \\ & & 1 \end{pmatrix} = 4\,\frac{N-1}{N^4}\,\sigma_1^4\sigma_2^2\sigma_3^2\,[(N-1)\,\rho_{23}^2 + 8\rho_{12}\rho_{13}\rho_{23} + 2\,(N+1)\,\rho_{12}^2\rho_{13}^2]$,

(44) $\mu\begin{pmatrix} 2 & \cdot & \cdot \\ & 1 & 1 \\ & & \cdot \end{pmatrix} = 4\,\frac{N-1}{N^4}\,\sigma_1^4\sigma_2^3\sigma_3\,[(N-1)\,\rho_{23} + 4\rho_{12}\,(\rho_{13} + \rho_{12}\rho_{23}) + 2\,(N+1)\,\rho_{12}^3\rho_{13}]$,

(45) $\mu\begin{pmatrix} 2 & \cdot & \cdot \\ & \cdot & 2 \\ & & \cdot \end{pmatrix} = 2\,\frac{N-1}{N^4}\,\sigma_1^4\sigma_2^2\sigma_3^2\,[(N-1)\,(1 + \rho_{23}^2) + 4\,(\rho_{12}^2 + \rho_{13}^2) + 8\rho_{12}\rho_{13}\rho_{23}$
$\qquad + 4\,(N+1)\,\rho_{12}^2\rho_{13}^2]$,

(46) $\mu\begin{pmatrix} 2 & 1 & 1 \\ & \cdot & \cdot \\ & & \cdot \end{pmatrix} = 2\,\frac{(N-1)(N+3)}{N^4}\,\sigma_1^6\sigma_2\sigma_3\,(\rho_{23} + 5\rho_{12}\rho_{13})$,

(47) $\mu\begin{pmatrix} 2 & \cdot & 1 \\ & \cdot & 1 \\ & & \cdot \end{pmatrix} = 2\,\frac{(N-1)(N+3)}{N^4}\,\sigma_1^5\sigma_2\sigma_3^2\,(\rho_{12} + \rho_{13}\rho_{23} + 4\rho_{12}\rho_{13}^2)$,

(48) $\mu\begin{pmatrix} 1 & \cdot & \cdot \\ & 1 & 1 \\ & & 1 \end{pmatrix} = 4\,\frac{N-1}{N^4}\,\sigma_1^2\sigma_2^3\sigma_3^3\,[2\rho_{12}\rho_{13} + (N+1)\,\rho_{23}\,(\rho_{12}^2 + \rho_{13}^2) + (N+5)\,\rho_{12}\rho_{13}\rho_{23}^2]$,

(49) $\mu\begin{pmatrix} 1 & \cdot & \cdot \\ & 1 & 2 \\ & & \cdot \end{pmatrix} = 2\,\frac{N-1}{N^4}\,\sigma_1^2\sigma_2^4\sigma_3^2\,[(N+3)\,\rho_{12}^2 + 2\rho_{13}^2 + 4\,(N+2)\,\rho_{12}\rho_{13}\rho_{23}$
$\qquad + (N+5)\,\rho_{12}^2\rho_{23}^2]$,

(50) $\mu\begin{pmatrix} 1 & 1 & 1 \\ & 1 & \cdot \\ & & \cdot \end{pmatrix} = 2\,\frac{N-1}{N^4}\,\sigma_1^4\sigma_2^3\sigma_3\,[2\rho_{23} + 2\,(N+2)\,\rho_{12}\rho_{13} + (3N+7)\,\rho_{12}^2\rho_{23} + (N+5)\,\rho_{12}^3\rho_{13}]$,

(51) $\mu\begin{pmatrix} 1 & \cdot & 1 \\ & 1 & 1 \\ & & \cdot \end{pmatrix} = 2\,\frac{N-1}{N^4}\,\sigma_1^3\sigma_2^3\sigma_3^2\,[2\,(\rho_{12} + N\rho_{13}\rho_{23}) + 4\rho_{12}\,(\rho_{13}^2 + \rho_{23}^2) + (N+1)\,\rho_{12}^3$
$\qquad + (3N+7)\,\rho_{12}^2\rho_{13}\rho_{23}]$,

(52) $\mu\begin{pmatrix} 1 & \cdot & \cdot \\ & \cdot & 3 \\ & & \cdot \end{pmatrix} = 6\,\frac{N-1}{N^4}\,\sigma_1^2\sigma_2^3\sigma_3^3\,[(N+1)\,\rho_{12}\rho_{13}\,(1 + \rho_{23}^2) + 2\rho_{23}\,(\rho_{12}^2 + \rho_{13}^2)]$,

(53) $\mu\begin{pmatrix} 1 & 1 & 2 \\ & \cdot & \cdot \\ & & \cdot \end{pmatrix} = 2\,\frac{(N-1)(N+3)}{N^4}\,\sigma_1^5\sigma_2\sigma_3^2\,(\rho_{12} + 2\rho_{13}\rho_{23} + 3\rho_{12}\rho_{13}^2)$,

(54) $\mu\begin{pmatrix} 1 & \cdot & 1 \\ & \cdot & 2 \\ & & \cdot \end{pmatrix} = 2\,\frac{N-1}{N^4}\,\sigma_1^3\sigma_2^2\sigma_3^3\,[(N+1)\,\rho_{13}\,(1 + \rho_{23}^2) + 4\rho_{12}\rho_{23} + 2\rho_{13}^3 + 2\,(N+2)\,\rho_{12}^2\rho_{13}$
$\qquad + 2\,(N+3)\,\rho_{12}\rho_{13}^2\rho_{23}]$,

(55) $\mu\begin{pmatrix} 1 & 1 & 1 \\ & \cdot & 1 \\ & & \cdot \end{pmatrix} = 2\,\frac{N-1}{N^4}\,\sigma_1^4\sigma_2^2\sigma_3^2\,[1 + \rho_{23}^2 + (N+2)\,(\rho_{12}^2 + \rho_{13}^2) + (3N+7)\,\rho_{12}\rho_{13}\rho_{23}$
$\qquad + (N+5)\,\rho_{12}^2\rho_{13}^2]$,

(56) $\mu\begin{pmatrix} \cdot & \cdot & 1 \\ & \cdot & 3 \\ & & \cdot \end{pmatrix} = 3\,\frac{N-1}{N^4}\,\sigma_1\sigma_2^3\sigma_3^4\,[(N+1)\,(\rho_{12} + \rho_{13}\rho_{23}^2) + (N+5)\,\rho_{23}\,(\rho_{13} + \rho_{12}\rho_{23})]$,

(57) $\mu\begin{pmatrix} \cdot & \cdot & 2 \\ & \cdot & 2 \\ & & \cdot \end{pmatrix} = \frac{N-1}{N^4}\,\sigma_1^2\sigma_2^2\sigma_3^4\,[(N+1)\,(1 + 2\rho_{12}^2 + 3\rho_{13}^2\rho_{23}^2)$
$\qquad + (N+5)\,(\rho_{13}^2 + \rho_{23}^2 + 4\rho_{12}\rho_{13}\rho_{23})]$,

(58) $\mu\begin{pmatrix} \cdot & 1 & 1 \\ & \cdot & 2 \\ & & \cdot \end{pmatrix} = \frac{N-1}{N^4}\,\sigma_1^2\sigma_2^3\sigma_3^3\,[(N+5)\,\rho_{23} + (3N+7)\,\rho_{12}\rho_{13} + (N+1)\,\rho_{23}^3$
$\qquad + 2\,(N+3)\,\rho_{23}\,(\rho_{12}^2 + \rho_{13}^2) + (3N+11)\,\rho_{12}\rho_{13}\rho_{23}^2]$.

4 *variates*

(59) $\mu\begin{pmatrix} 2 & \cdot & \cdot & \cdot \\ & 1 & \cdot & \cdot \\ & & \cdot & 1 \\ & & & \cdot \end{pmatrix} = 4\,\frac{N-1}{N^4}\,\sigma_1^4\sigma_2^2\sigma_3\sigma_4\,[(N-1)\,\rho_{23}\rho_{24} + 4\rho_{12}\,(\rho_{13}\rho_{24} + \rho_{14}\rho_{23})$
$\qquad + 2\,(N+1)\,\rho_{12}^2\rho_{13}\rho_{14}]$,

(60) $\mu\begin{pmatrix} 2 & 1 & \cdot & \cdot \\ & \cdot & \cdot & \cdot \\ & & \cdot & 1 \\ & & & \cdot \end{pmatrix} = 2\,\frac{(N-1)(N+3)}{N^4}\,\sigma_1^5\sigma_2\sigma_3\sigma_4\,(\rho_{13}\rho_{24} + \rho_{14}\rho_{23} + 4\rho_{12}\rho_{13}\rho_{14})$,

$$(61)\quad \mu\begin{pmatrix}2&\cdot&\cdot&\cdot\\&\cdot&1&1\\&&\cdot&\cdot\\&&&\cdot\end{pmatrix}=2\,\frac{N-1}{N^4}\,\sigma_1^4\sigma_2^2\sigma_3\sigma_4\big[(N-1)(\rho_{34}+\rho_{23}\rho_{24})+4\rho_{12}(\rho_{12}\rho_{34}+\rho_{13}\rho_{24}+\rho_{14}\rho_{23})$$
$$+\,4\rho_{13}\rho_{14}\{1+(N+1)\rho_{12}^2\}\big],$$

$$(62)\quad \mu\begin{pmatrix}1&\cdot&\cdot&\cdot\\&1&\cdot&\cdot\\&&1&\cdot\\&&&1\end{pmatrix}=4\,\frac{N-1}{N^4}\,\sigma_1^2\sigma_2^2\sigma_3^2\sigma_4^2\big[(N-1)(\rho_{12}^2\rho_{34}^2+\rho_{13}^2\rho_{24}^2+\rho_{14}^2\rho_{23}^2)$$
$$+\,4(\rho_{12}\rho_{13}\rho_{24}\rho_{34}+\rho_{12}\rho_{14}\rho_{23}\rho_{34}+\rho_{13}\rho_{14}\rho_{23}\rho_{24})\big],$$

$$(63)\quad \mu\begin{pmatrix}1&\cdot&\cdot&1\\&1&\cdot&\cdot\\&&1&\cdot\\&&&\cdot\end{pmatrix}=4\,\frac{N-1}{N^4}\,\sigma_1^3\sigma_2^2\sigma_3^2\sigma_4\big[(\rho_{12}\rho_{34}+\rho_{13}\rho_{24})\{2\rho_{23}+(N+1)\rho_{12}\rho_{13}\}$$
$$+\,\rho_{14}\rho_{23}\{(N-1)\rho_{23}+4\rho_{12}\rho_{13}\}\big],$$

$$(64)\quad \mu\begin{pmatrix}1&1&\cdot&\cdot\\&1&\cdot&\cdot\\&&\cdot&1\\&&&\cdot\end{pmatrix}=2\,\frac{N-1}{N^4}\,\sigma_1^3\sigma_2^3\sigma_3\sigma_4\big[2(N+1)\rho_{12}(\rho_{13}\rho_{14}+\rho_{23}\rho_{24})$$
$$+\,(\rho_{13}\rho_{24}+\rho_{14}\rho_{23})\{2+(N+5)\rho_{12}^2\}\big],$$

$$(65)\quad \mu\begin{pmatrix}1&\cdot&1&1\\&1&\cdot&\cdot\\&&\cdot&\cdot\\&&&\cdot\end{pmatrix}=2\,\frac{N-1}{N^4}\,\sigma_1^4\sigma_2^2\sigma_3\sigma_4\big[2\rho_{23}\rho_{24}+2(N+2)\rho_{12}(\rho_{13}\rho_{24}+\rho_{14}\rho_{23})$$
$$+\,\rho_{12}^2\{(N+3)\rho_{34}+(N+5)\rho_{13}\rho_{14}\}\big],$$

$$(66)\quad \mu\begin{pmatrix}1&\cdot&1&\cdot\\&1&\cdot&\cdot\\&&\cdot&1\\&&&\cdot\end{pmatrix}=2\,\frac{N-1}{N^4}\,\sigma_1^3\sigma_2^2\sigma_3^2\sigma_4\big[2\rho_{23}(\rho_{12}\rho_{34}+\rho_{14}\rho_{23})+2\rho_{12}\rho_{24}(1+2\rho_{13}^2)+2N\rho_{13}\rho_{23}\rho_{24}$$
$$+\,(N+1)\rho_{12}^2(\rho_{14}+\rho_{13}\rho_{34})+2(N+3)\rho_{12}\rho_{13}\rho_{14}\rho_{23}\big],$$

$$(67)\quad \mu\begin{pmatrix}1&\cdot&\cdot&1\\&1&1&\cdot\\&&\cdot&\cdot\\&&&\cdot\end{pmatrix}=2\,\frac{N-1}{N^4}\,\sigma_1^3\sigma_2^3\sigma_3\sigma_4\big[2\{\rho_{12}\rho_{34}+\rho_{13}\rho_{24}+(N-1)\rho_{14}\rho_{23}\}+4\rho_{12}\rho_{13}\rho_{14}$$
$$+\,(N+1)\rho_{12}^2(\rho_{13}\rho_{34}+3\rho_{13}\rho_{24})+4\rho_{12}\rho_{23}(\rho_{24}+\rho_{12}\rho_{14})\big],$$

$$(68)\quad \mu\begin{pmatrix}1&\cdot&\cdot&\cdot\\&1&\cdot&\cdot\\&&\cdot&2\\&&&\cdot\end{pmatrix}=2\,\frac{N-1}{N^4}\,\sigma_1^2\sigma_2^2\sigma_3^2\sigma_4^2\big[(N-1)\rho_{12}^2(1+\rho_{34}^2)+2(\rho_{13}^2\rho_{24}^2+\rho_{14}^2\rho_{23}^2)$$
$$+\,4\rho_{12}\{\rho_{13}\rho_{23}+\rho_{14}\rho_{24}+\rho_{34}(\rho_{13}\rho_{24}+\rho_{14}\rho_{23})\}+4N\rho_{13}\rho_{14}\rho_{23}\rho_{24}\big],$$

$$(69)\quad \mu\begin{pmatrix}1&2&\cdot&\cdot\\&\cdot&\cdot&\cdot\\&&\cdot&1\\&&&\cdot\end{pmatrix}=2\,\frac{N-1}{N^4}\,\sigma_1^4\sigma_2^2\sigma_3\sigma_4\big[(N+3)\rho_{13}\rho_{14}+2\rho_{23}\rho_{24}+2(N+2)\rho_{12}(\rho_{13}\rho_{24}+\rho_{14}\rho_{23})$$
$$+\,(N+5)\rho_{12}^2\rho_{13}\rho_{14}\big],$$

$$(70)\quad \mu\begin{pmatrix}1&\cdot&\cdot&1\\&\cdot&2&\cdot\\&&\cdot&\cdot\\&&&\cdot\end{pmatrix}=2\,\frac{N-1}{N^4}\,\sigma_1^3\sigma_2^2\sigma_3^2\sigma_4\big[(N-1)\rho_{14}(1+\rho_{23}^2)+2(\rho_{12}\rho_{24}+\rho_{13}\rho_{34})+2\rho_{14}(\rho_{12}^2+\rho_{13}^2)$$
$$+\,2(\rho_{12}\rho_{34}+\rho_{13}\rho_{24})\{\rho_{23}+(N+1)\rho_{12}\rho_{13}\}+4\rho_{12}\rho_{13}\rho_{14}\rho_{23}\big],$$

$$(71)\quad \mu\begin{pmatrix}1&\cdot&\cdot&\cdot\\&\cdot&2&1\\&&\cdot&\cdot\\&&&\cdot\end{pmatrix}=2\,\frac{N-1}{N^4}\,\sigma_1^2\sigma_2^3\sigma_3^2\sigma_4\big[2\rho_{24}(\rho_{12}^2+\rho_{13}^2)+4(\rho_{12}\rho_{14}\rho_{23}+\rho_{12}^2\rho_{23}\rho_{34})$$
$$+\,(N+1)\rho_{12}\{\rho_{14}(1+\rho_{23}^2)+2\rho_{13}(\rho_{34}+\rho_{23}\rho_{24})\}\big],$$

$$(72)\quad \mu\begin{pmatrix}1&1&1&1\\&\cdot&\cdot&\cdot\\&&\cdot&\cdot\\&&&\cdot\end{pmatrix}=2\,\frac{(N-1)(N+3)}{N^4}\,\sigma_1^5\sigma_2\sigma_3\sigma_4(\rho_{12}\rho_{34}+\rho_{13}\rho_{24}+\rho_{14}\rho_{23}+3\rho_{12}\rho_{13}\rho_{14}),$$

$$(73)\quad \mu\begin{pmatrix}1&1&1&\cdot\\&\cdot&\cdot&1\\&&\cdot&\cdot\\&&&\cdot\end{pmatrix}=2\,\frac{N-1}{N^4}\,\sigma_1^4\sigma_2^2\sigma_3\sigma_4\big[\rho_{34}+\rho_{23}\rho_{24}+(N+2)\{\rho_{13}(\rho_{14}+\rho_{12}\rho_{24})+\rho_{12}^2\rho_{34}\}$$
$$+\,(N+5)\rho_{12}^2\rho_{13}\rho_{14}+(2N+5)\rho_{12}\rho_{14}\rho_{23}\big],$$

$$(74)\quad \mu\begin{pmatrix}1&1&\cdot&\cdot\\&\cdot&1&1\\&&\cdot&\cdot\\&&&\cdot\end{pmatrix}=2\,\frac{N-1}{N^4}\,\sigma_1^3\sigma_2^3\sigma_3\sigma_4\big[(N+1)\rho_{12}(\rho_{34}+\rho_{23}\rho_{24})+2(N+2)\rho_{12}\rho_{13}\rho_{14}$$
$$+\,2(\rho_{13}\rho_{24}+\rho_{14}\rho_{23}+\rho_{12}^3\rho_{34})+(N+3)\rho_{12}^2(\rho_{13}\rho_{24}+\rho_{14}\rho_{23})\big],$$

$$(75)\quad \mu\begin{pmatrix}1&1&\cdot&\cdot\\&\cdot&1&\cdot\\&&\cdot&1\\&&&\cdot\end{pmatrix}=2\,\frac{N-1}{N^4}\,\sigma_1^3\sigma_2^2\sigma_3^2\sigma_4\big[\rho_{12}(\rho_{34}+3\rho_{23}\rho_{24})+\rho_{14}\{1+2\rho_{12}^2+\rho_{23}^2+(N+1)\rho_{13}^2$$
$$+\,2(N+2)\rho_{12}\rho_{13}\rho_{23}\}+N\rho_{12}(\rho_{24}+\rho_{23}\rho_{34})+\rho_{12}\rho_{13}\{2\rho_{12}\rho_{34}+(N+3)\rho_{13}\rho_{24}\}\big],$$

$$(76)\quad \mu\begin{pmatrix}1 & \cdot & \cdot & \cdot \\ & \cdot & 1 & 1 \\ & & \cdot & 1 \\ & & & \cdot\end{pmatrix} = 2\frac{N-1}{N^4}\sigma_1^2\sigma_2^2\sigma_3^2\sigma_4^2[\rho_{12}^2+\rho_{13}^2+\rho_{14}^2+\rho_{12}^2\rho_{34}^2+\rho_{13}^2\rho_{24}^2+\rho_{14}^2\rho_{23}^2$$
$$+\rho_{12}\rho_{13}\{(N+1)\rho_{23}+(N+3)\rho_{24}\rho_{34}\}+\rho_{12}\rho_{14}\{(N+1)\rho_{24}+(N+3)\rho_{23}\rho_{34}\}$$
$$+\rho_{13}\rho_{14}\{(N+1)\rho_{34}+(N+3)\rho_{23}\rho_{24}\}],$$

$$(77)\quad \mu\begin{pmatrix}\cdot & 3 & \cdot & \cdot \\ & \cdot & \cdot & \cdot \\ & & \cdot & 1 \\ & & & \cdot\end{pmatrix} = 3\frac{N-1}{N^4}\sigma_1^3\sigma_2^3\sigma_3\sigma_4[4\rho_{12}(\rho_{13}\rho_{14}+\rho_{23}\rho_{24})+(N+1)(1+\rho_{12}^2)(\rho_{13}\rho_{24}+\rho_{14}\rho_{23})],$$

$$(78)\quad \mu\begin{pmatrix}\cdot & 2 & 1 & 1 \\ & \cdot & \cdot & \cdot \\ & & \cdot & \cdot \\ & & & \cdot\end{pmatrix} = \frac{N-1}{N^4}\sigma_1^4\sigma_2^2\sigma_3\sigma_4[(N+1)(\rho_{34}+2\rho_{23}\rho_{24}+3\rho_{12}^2\rho_{13}\rho_{14})$$
$$+(N+5)(\rho_{13}\rho_{14}+\rho_{12}^2\rho_{34}+2\rho_{12}\rho_{13}\rho_{24}+2\rho_{12}\rho_{23}\rho_{14})],$$

$$(79)\quad \mu\begin{pmatrix}\cdot & 2 & 1 & \cdot \\ & \cdot & \cdot & 1 \\ & & \cdot & \cdot \\ & & & \cdot\end{pmatrix} = \frac{N-1}{N^4}\sigma_1^3\sigma_2^3\sigma_3\sigma_4[(N+5)\rho_{12}\rho_{34}+4\rho_{13}\rho_{24}$$
$$+(N+1)(3\rho_{14}\rho_{23}+2\rho_{12}^2\rho_{13}\rho_{24}+\rho_{12}^3\rho_{34})$$
$$+2(N+3)\rho_{12}(\rho_{13}\rho_{14}+\rho_{23}\rho_{24})+(N+9)\rho_{12}^2\rho_{14}\rho_{23}],$$

$$(80)\quad \mu\begin{pmatrix}\cdot & 2 & 1 & \cdot \\ & \cdot & \cdot & \cdot \\ & & \cdot & 1 \\ & & & \cdot\end{pmatrix} = \frac{N-1}{N^4}\sigma_1^3\sigma_2^2\sigma_3^2\sigma_4[4\rho_{12}(\rho_{24}+\rho_{23}\rho_{34})+4\rho_{13}^2\rho_{14}$$
$$+(N+1)\{(1+\rho_{12}^2)(\rho_{14}+\rho_{13}\rho_{34})+2(\rho_{14}\rho_{23}^2+\rho_{12}\rho_{13}^2\rho_{24})\}$$
$$+2(N+3)\rho_{13}\rho_{23}\rho_{24}+2(N+5)\rho_{12}\rho_{13}\rho_{14}\rho_{23}],$$

$$(81)\quad \mu\begin{pmatrix}\cdot & 2 & \cdot & \cdot \\ & \cdot & \cdot & \cdot \\ & & \cdot & 2 \\ & & & \cdot\end{pmatrix} = \frac{N-1}{N^4}\sigma_1^2\sigma_2^2\sigma_3^2\sigma_4^2[(N-1)(1+\rho_{12}^2)(1+\rho_{34}^2)+2(\rho_{13}^2+\rho_{14}^2+\rho_{23}^2+\rho_{24}^2)$$
$$+2N(\rho_{13}^2\rho_{24}^2+\rho_{14}^2\rho_{23}^2)+4\{\rho_{12}\rho_{13}\rho_{23}+\rho_{12}\rho_{14}\rho_{24}+\rho_{13}\rho_{14}\rho_{34}$$
$$+\rho_{23}\rho_{24}\rho_{34}+\rho_{12}\rho_{34}(\rho_{13}\rho_{24}+\rho_{14}\rho_{23})+(N+2)\rho_{13}\rho_{14}\rho_{23}\rho_{24}\}],$$

$$(82)\quad \mu\begin{pmatrix}\cdot & 1 & 1 & 1 \\ & \cdot & 1 & \cdot \\ & & \cdot & \cdot \\ & & & \cdot\end{pmatrix} = \frac{N-1}{N^4}\sigma_1^3\sigma_2^2\sigma_3^2\sigma_4[\rho_{14}\{2(1+\rho_{23}^2)+(N+1)(\rho_{12}^2+\rho_{13}^2)\}$$
$$+2(N+2)\rho_{23}(\rho_{12}\rho_{34}+\rho_{13}\rho_{24})+(N+3)(\rho_{12}\rho_{24}+\rho_{13}\rho_{34}+2\rho_{12}\rho_{13}\rho_{14}\rho_{23})$$
$$+(N+5)\rho_{12}\rho_{13}(\rho_{12}\rho_{34}+\rho_{13}\rho_{24})],$$

$$(83)\quad \mu\begin{pmatrix}\cdot & 1 & 1 & \cdot \\ & \cdot & \cdot & 1 \\ & & \cdot & 1 \\ & & & \cdot\end{pmatrix} = \frac{N-1}{N^4}\sigma_1^2\sigma_2^2\sigma_3^2\sigma_4^2[1+\rho_{12}^2+\rho_{13}^2+\rho_{24}^2+\rho_{34}^2+N(\rho_{14}^2+\rho_{23}^2)$$
$$+\rho_{12}^2\rho_{34}^2+\rho_{13}^2\rho_{24}^2+(N+2)\rho_{14}^2\rho_{23}^2$$
$$+(N+3)(\rho_{12}\rho_{13}\rho_{23}+\rho_{12}\rho_{14}\rho_{24}+\rho_{13}\rho_{14}\rho_{34}+\rho_{23}\rho_{24}\rho_{34})$$
$$+(N+7)\rho_{14}\rho_{23}(\rho_{12}\rho_{34}+\rho_{13}\rho_{24})+(3N+1)\rho_{12}\rho_{13}\rho_{24}\rho_{34}].$$

5 variates

$$(84)\quad \mu\begin{pmatrix}2 & \cdot & \cdot & \cdot & \cdot \\ & \cdot & 1 & \cdot & \cdot \\ & & \cdot & \cdot & \cdot \\ & & & \cdot & 1 \\ & & & & \cdot\end{pmatrix} = 2\frac{N-1}{N^4}\sigma_1^4\sigma_2\sigma_3\sigma_4\sigma_5[(N-1)(\rho_{24}\rho_{35}+\rho_{25}\rho_{34})+4\rho_{12}(\rho_{14}\rho_{35}+\rho_{15}\rho_{34})$$
$$+4\rho_{13}(\rho_{14}\rho_{25}+\rho_{15}\rho_{24})+4(N+1)\rho_{12}\rho_{13}\rho_{14}\rho_{15}],$$

$$(85)\quad \mu\begin{pmatrix}1 & \cdot & \cdot & \cdot & \cdot \\ & 1 & \cdot & \cdot & \cdot \\ & & 1 & \cdot & \cdot \\ & & & \cdot & 1 \\ & & & & \cdot\end{pmatrix} = 4\frac{N-1}{N^4}\sigma_1^2\sigma_2^2\sigma_3^2\sigma_4\sigma_5[(N-1)(\rho_{12}^2\rho_{34}\rho_{35}+\rho_{13}^2\rho_{24}\rho_{25}+\rho_{23}^2\rho_{14}\rho_{15})$$
$$+2\rho_{12}\rho_{13}(\rho_{24}\rho_{35}+\rho_{25}\rho_{34})+2\rho_{12}\rho_{23}(\rho_{14}\rho_{35}+\rho_{15}\rho_{34})$$
$$+2\rho_{13}\rho_{23}(\rho_{14}\rho_{25}+\rho_{15}\rho_{24})],$$

$$(86)\quad \mu\begin{pmatrix}1 & \cdot & \cdot & \cdot & \cdot \\ & 1 & 1 & \cdot & \cdot \\ & & \cdot & \cdot & \cdot \\ & & & \cdot & 1 \\ & & & & \cdot\end{pmatrix} = 2\frac{N-1}{N^4}\sigma_1^2\sigma_2^3\sigma_3\sigma_4\sigma_5[2\rho_{12}(\rho_{14}\rho_{35}+\rho_{15}\rho_{34})+2(\rho_{13}+2\rho_{12}\rho_{23})(\rho_{14}\rho_{25}+\rho_{15}\rho_{24})$$
$$+2(N-1)\rho_{14}\rho_{15}\rho_{23}+(N+1)\rho_{12}\{\rho_{12}(\rho_{24}\rho_{35}+\rho_{25}\rho_{34})+2\rho_{13}\rho_{24}\rho_{25}\}],$$

$$(87)\quad \mu\begin{pmatrix} 1 & \cdot & \cdot & \cdot & \cdot \\ & 1 & \cdot & \cdot & \cdot \\ & & \cdot & 1 & 1 \\ & & & \cdot & \cdot \\ & & & & \cdot \end{pmatrix} = 2\,\frac{N-1}{N^4}\,\sigma_1{}^2\sigma_2{}^2\sigma_3{}^2\sigma_4\sigma_5\,[(N-1)\,\rho_{12}{}^2\,(\rho_{45}+\rho_{34}\rho_{35}) + 2\,(\rho_{12}{}^2\rho_{34}\rho_{35} + \rho_{23}{}^2\rho_{14}\rho_{15})$$
$$+\, 2\rho_{12}\rho_{13}\,(\rho_{24}\rho_{35}+\rho_{25}\rho_{34}) + 2\rho_{12}\rho_{23}\,(2\rho_{13}\rho_{45}+\rho_{14}\rho_{35}+\rho_{15}\rho_{34})$$
$$+\, 2\,(\rho_{12}+N\rho_{13}\rho_{23})\,(\rho_{14}\rho_{25}+\rho_{15}\rho_{24})],$$

$$(88)\quad \mu\begin{pmatrix} 1 & \cdot & \cdot & \cdot & \cdot \\ & \cdot & 2 & \cdot & \cdot \\ & & \cdot & \cdot & \cdot \\ & & & \cdot & 1 \\ & & & & \cdot \end{pmatrix} = 2\,\frac{N-1}{N^4}\,\sigma_1{}^2\sigma_2{}^2\sigma_3{}^2\sigma_4\sigma_5\,[(N-1)\,\rho_{14}\rho_{15}\,(1+\rho_{23}{}^2) + 2\,(\rho_{12}{}^2\rho_{34}\rho_{35} + \rho_{13}{}^2\rho_{24}\rho_{25})$$
$$+\, 2N\rho_{12}\rho_{13}\,(\rho_{24}\rho_{35}+\rho_{25}\rho_{34}) + 2\rho_{12}\rho_{14}\,(\rho_{25}+\rho_{23}\rho_{35}) + 2\rho_{12}\rho_{15}\,(\rho_{24}+\rho_{23}\rho_{34})$$
$$+\, 2\rho_{13}\rho_{14}\,(\rho_{35}+\rho_{23}\rho_{25}) + 2\rho_{13}\rho_{15}\,(\rho_{34}+\rho_{23}\rho_{24})],$$

$$(89)\quad \mu\begin{pmatrix} 1 & 1 & 1 & \cdot & \cdot \\ & \cdot & \cdot & \cdot & \cdot \\ & & \cdot & \cdot & \cdot \\ & & & \cdot & 1 \\ & & & & \cdot \end{pmatrix} = 2\,\frac{N-1}{N^4}\,\sigma_1{}^4\sigma_2\sigma_3\sigma_4\sigma_5\,[\rho_{24}\rho_{35}+\rho_{25}\rho_{34}+2\rho_{12}\rho_{13}\rho_{14}\rho_{15}$$
$$+\, (N+2)\,\{\rho_{12}\,(\rho_{14}\rho_{35}+\rho_{15}\rho_{34}) + \rho_{13}\,(\rho_{14}\rho_{25}+\rho_{15}\rho_{24})\}$$
$$+\, (N+3)\,\rho_{14}\rho_{15}\,(\rho_{23}+\rho_{12}\rho_{13})],$$

$$(90)\quad \mu\begin{pmatrix} 1 & 1 & \cdot & \cdot & \cdot \\ & \cdot & \cdot & \cdot & \cdot \\ & & \cdot & 1 & 1 \\ & & & \cdot & \cdot \\ & & & & \cdot \end{pmatrix} = 2\,\frac{N-1}{N^4}\,\sigma_1{}^3\sigma_2\sigma_3{}^2\sigma_4\sigma_5\,[\rho_{14}\rho_{35}+\rho_{15}\rho_{34}+2\rho_{13}\rho_{14}\rho_{15}+(N-1)\,\rho_{12}\,(\rho_{45}+\rho_{34}\rho_{35})$$
$$+\, \rho_{13}\,(\rho_{23}\rho_{45}+\rho_{24}\rho_{35}+\rho_{25}\rho_{34}) + (\rho_{23}+2\rho_{12}\rho_{13})\,(\rho_{13}\rho_{45}+\rho_{14}\rho_{35}+\rho_{15}\rho_{34})$$
$$+\, (N+1)\,\{\rho_{13}{}^2\,(\rho_{14}\rho_{25}+\rho_{15}\rho_{24}) + 2\rho_{13}\rho_{14}\rho_{15}\rho_{23}\}],$$

$$(91)\quad \mu\begin{pmatrix} 1 & \cdot & 1 & \cdot & \cdot \\ & \cdot & 1 & \cdot & \cdot \\ & & \cdot & \cdot & \cdot \\ & & & \cdot & 1 \\ & & & & \cdot \end{pmatrix} = 2\,\frac{N-1}{N^4}\,\sigma_1{}^2\sigma_2\sigma_3{}^2\sigma_4\sigma_5\,[(1+2\rho_{13}{}^2)\,(\rho_{14}\rho_{25}+\rho_{15}\rho_{24})$$
$$+\, 2\rho_{12}\rho_{34}\rho_{35} + \rho_{23}\,(\rho_{14}\rho_{35}+\rho_{15}\rho_{34}) + N\rho_{13}\,(\rho_{24}\rho_{35}+\rho_{25}\rho_{34})$$
$$+\, (N+3)\,\rho_{12}\rho_{13}\,(\rho_{14}\rho_{35}+\rho_{15}\rho_{34}) + (N+1)\,\rho_{14}\rho_{15}\,(\rho_{12}+\rho_{13}\rho_{23})],$$

$$(92)\quad \mu\begin{pmatrix} 1 & \cdot & \cdot & \cdot & \cdot \\ & \cdot & 1 & \cdot & \cdot \\ & & \cdot & 1 & 1 \\ & & & \cdot & \cdot \\ & & & & \cdot \end{pmatrix} = 2\,\frac{N-1}{N^4}\,\sigma_1{}^2\sigma_2\sigma_3{}^2\sigma_4\sigma_5\,[(N-1)\,\rho_{12}\,(\rho_{45}+\rho_{34}\rho_{35}) + (N+1)\,\{\rho_{13}{}^2\,(\rho_{14}\rho_{25}+\rho_{15}\rho_{24})$$
$$+\, 2\rho_{13}\rho_{14}\rho_{15}\rho_{23}\} + \rho_{14}\rho_{25}+\rho_{15}\rho_{24}+2\rho_{12}\rho_{14}\rho_{15}+2\rho_{12}\rho_{13}\,(\rho_{14}\rho_{35}+\rho_{15}\rho_{34})$$
$$+\, 2\rho_{13}\rho_{45}\,(\rho_{23}+\rho_{12}\rho_{13}) + \rho_{34}\,(\rho_{13}\rho_{25}+\rho_{15}\rho_{23}) + \rho_{35}\,(\rho_{13}\rho_{24}+\rho_{14}\rho_{23})],$$

$$(93)\quad \mu\begin{pmatrix} 1 & \cdot & \cdot & \cdot & \cdot \\ & \cdot & 1 & \cdot & \cdot \\ & & \cdot & \cdot & 1 \\ & & & \cdot & 1 \\ & & & & \cdot \end{pmatrix} = 2\,\frac{N-1}{N^4}\,\sigma_1{}^2\sigma_2\sigma_3{}^2\sigma_4\sigma_5{}^2\,[\rho_{12}\rho_{14}\,(1+\rho_{35}{}^2) + \rho_{12}\rho_{15}\rho_{45} + \rho_{24}\,(\rho_{13}{}^2+\rho_{15}{}^2)$$
$$+\, \rho_{13}\rho_{23}\,(\rho_{14}+\rho_{15}\rho_{45}) + \rho_{13}\rho_{25}\,(\rho_{13}\rho_{45}+3\rho_{14}\rho_{35}) + \rho_{15}\rho_{34}\,(\rho_{15}\rho_{23}+3\rho_{12}\rho_{35})$$
$$+\, N\,\{\rho_{12}\rho_{13}\,(\rho_{34}+\rho_{35}\rho_{45}) + \rho_{14}\rho_{15}\,(\rho_{25}+\rho_{23}\rho_{35})\} + (N+1)\,\rho_{13}\rho_{15}\rho_{24}\rho_{35}$$
$$+\, (N+2)\,\rho_{13}\rho_{15}\rho_{25}\rho_{34}],$$

$$(94)\quad \mu\begin{pmatrix} \cdot & 2 & 1 & \cdot & \cdot \\ & \cdot & \cdot & \cdot & \cdot \\ & & \cdot & \cdot & \cdot \\ & & & \cdot & 1 \\ & & & & \cdot \end{pmatrix} = \frac{N-1}{N^4}\,\sigma_1{}^2\sigma_2{}^2\sigma_3\sigma_4\sigma_5\,[4\rho_{12}\,(\rho_{24}\rho_{35}+\rho_{25}\rho_{34}) + 4\rho_{13}\rho_{24}\rho_{25} + 4\rho_{14}\rho_{15}\,(\rho_{13}+2\rho_{12}\rho_{23})$$
$$+\, (N+1)\,\{(1+\rho_{12}{}^2)\,(\rho_{14}\rho_{35}+\rho_{15}\rho_{34}) + 2\,(\rho_{23}+\rho_{12}\rho_{13})\,(\rho_{14}\rho_{25}+\rho_{15}\rho_{24})\}],$$

$$(95)\quad \mu\begin{pmatrix} \cdot & 2 & \cdot & \cdot & \cdot \\ & \cdot & \cdot & \cdot & \cdot \\ & & \cdot & 1 & 1 \\ & & & \cdot & \cdot \\ & & & & \cdot \end{pmatrix} = \frac{N-1}{N^4}\,\sigma_1{}^2\sigma_2{}^2\sigma_3{}^2\sigma_4\sigma_5\,[(N-1)\,(1+\rho_{12}{}^2)\,(\rho_{45}+\rho_{34}\rho_{35}) + 2N\,(\rho_{13}{}^2\rho_{24}\rho_{25}+\rho_{23}{}^2\rho_{14}\rho_{15})$$
$$+\, 2\,(N+2)\,\rho_{13}\rho_{23}\,(\rho_{14}\rho_{25}+\rho_{15}\rho_{24}) + 2\,(\rho_{14}\rho_{15}+\rho_{24}\rho_{25}) + 2\rho_{45}\,(\rho_{13}{}^2+\rho_{23}{}^2)$$
$$+\, 2\rho_{12}\,(\rho_{14}\rho_{25}+\rho_{15}\rho_{24}) + 2\rho_{13}\,(\rho_{14}\rho_{35}+\rho_{15}\rho_{34}) + 2\rho_{23}\,(\rho_{24}\rho_{35}+\rho_{25}\rho_{34})$$
$$+\, 2\rho_{12}\rho_{13}\,(\rho_{23}\rho_{45}+\rho_{24}\rho_{35}+\rho_{25}\rho_{34}) + 2\rho_{12}\rho_{23}\,(\rho_{13}\rho_{45}+\rho_{14}\rho_{35}+\rho_{15}\rho_{34})],$$

$$(96)\quad \mu\begin{pmatrix} \cdot & 1 & 1 & 1 & 1 \\ & \cdot & \cdot & \cdot & \cdot \\ & & \cdot & \cdot & \cdot \\ & & & \cdot & \cdot \\ & & & & \cdot \end{pmatrix} = \frac{N-1}{N^4}\,\sigma_1{}^4\sigma_2\sigma_3\sigma_4\sigma_5\,[(N+1)\,(\rho_{23}\rho_{45}+\rho_{24}\rho_{35}+\rho_{25}\rho_{34}+3\rho_{12}\rho_{13}\rho_{14}\rho_{15})$$
$$+\, (N+5)\,\{\rho_{12}\,(\rho_{13}\rho_{45}+\rho_{14}\rho_{35}) + \rho_{14}\,(\rho_{13}\rho_{25}+\rho_{15}\rho_{23}) + \rho_{15}\,(\rho_{12}\rho_{34}+\rho_{13}\rho_{24})\}],$$

$$(97)\quad \mu\begin{pmatrix} \cdot & 1 & 1 & 1 & \cdot \\ & \cdot & \cdot & \cdot & \cdot \\ & & \cdot & \cdot & 1 \\ & & & \cdot & \cdot \\ & & & & \cdot \end{pmatrix} = \frac{N-1}{N^4}\sigma_1{}^2\sigma_2\sigma_3{}^2\sigma_4\sigma_5[2\rho_{13}(\rho_{45}+\rho_{34}\rho_{35}) + 2\rho_{14}(\rho_{35}+\rho_{33}\rho_{45}) + 4\rho_{13}{}^2\rho_{15}\rho_{34}$$
$$+ (N+1)\{\rho_{13}{}^2(\rho_{12}\rho_{45}+\rho_{14}\rho_{25}) + 2\rho_{15}\rho_{23}\rho_{34} + (\rho_{15}+\rho_{13}\rho_{35})(\rho_{24}+\rho_{12}\rho_{14})\}$$
$$+ (N+3)\rho_{13}(\rho_{23}\rho_{45}+\rho_{25}\rho_{34}) + (N+5)\rho_{12}\rho_{15}(\rho_{13}\rho_{34}+\rho_{14}\rho_{23})],$$

$$(98)\quad \mu\begin{pmatrix} \cdot & 1 & 1 & \cdot & \cdot \\ & \cdot & 1 & \cdot & \cdot \\ & & \cdot & \cdot & \cdot \\ & & & \cdot & 1 \\ & & & & \cdot \end{pmatrix} = \frac{N-1}{N^4}\sigma_1{}^2\sigma_2{}^2\sigma_3{}^2\sigma_4\sigma_5[2\{\rho_{14}\rho_{15}(1+\rho_{23}{}^2) + \rho_{24}\rho_{25}(1+\rho_{13}{}^2) + \rho_{34}\rho_{35}(1+\rho_{12}{}^2)\}$$
$$+ (\rho_{14}\rho_{25}+\rho_{15}\rho_{24})\{(N+1)\rho_{12} + (N+3)\rho_{13}\rho_{23}\}$$
$$+ (\rho_{14}\rho_{35}+\rho_{15}\rho_{34})\{(N+1)\rho_{13} + (N+3)\rho_{12}\rho_{23}\}$$
$$+ (\rho_{24}\rho_{35}+\rho_{25}\rho_{34})\{(N+1)\rho_{23} + (N+3)\rho_{12}\rho_{13}\}],$$

$$(99)\quad \mu\begin{pmatrix} \cdot & 1 & 1 & \cdot & \cdot \\ & \cdot & \cdot & 1 & \cdot \\ & & \cdot & \cdot & 1 \\ & & & \cdot & \cdot \\ & & & & \cdot \end{pmatrix} = \frac{N-1}{N^4}\sigma_1{}^2\sigma_2{}^2\sigma_3{}^2\sigma_4\sigma_5[\rho_{45}(1+\rho_{12}{}^2+\rho_{13}{}^2+N\rho_{23}{}^2) + \rho_{14}\rho_{15}\{N+(N+2)\rho_{23}{}^2\}$$
$$+ \rho_{24}\rho_{25}(1+\rho_{13}{}^2) + \rho_{34}\rho_{35}(1+\rho_{12}{}^2) + \rho_{23}\rho_{34}\rho_{35} + 2\rho_{13}\rho_{14}\rho_{23}\rho_{25}$$
$$+ 3\rho_{12}\rho_{14}(\rho_{25}+\rho_{23}\rho_{35}) + 3\rho_{13}\rho_{15}(\rho_{34}+\rho_{23}\rho_{24}) + 3\rho_{12}\rho_{23}(\rho_{13}\rho_{45}+\rho_{15}\rho_{34})$$
$$+ N\{\rho_{12}\rho_{13}(\rho_{23}\rho_{45}+\rho_{24}\rho_{35}+\rho_{25}\rho_{34}) + \rho_{12}\rho_{15}\rho_{24} + \rho_{13}\rho_{14}\rho_{35}\}$$
$$+ (N+1)\rho_{12}\rho_{34}(\rho_{13}\rho_{25}+\rho_{15}\rho_{23}) + (N+2)\rho_{23}\rho_{25}(\rho_{34}+\rho_{13}\rho_{14})],$$

$$(100)\quad \mu\begin{pmatrix} \cdot & 1 & 1 & \cdot & \cdot \\ & \cdot & \cdot & 1 & \cdot \\ & & \cdot & \cdot & \cdot \\ & & & \cdot & 1 \\ & & & & \cdot \end{pmatrix} = \frac{N-1}{N^4}\sigma_1{}^2\sigma_2{}^2\sigma_3\sigma_4{}^2\sigma_5[(1+\rho_{12}{}^2)(\rho_{35}+\rho_{34}\rho_{45}) + (\rho_{13}+3\rho_{12}\rho_{23})(\rho_{15}+\rho_{14}\rho_{45})$$
$$+ \rho_{24}{}^2(\rho_{35}+\rho_{13}\rho_{15}) + 3\rho_{24}(\rho_{25}\rho_{34}+\rho_{13}\rho_{14}\rho_{25})$$
$$+ N\{\rho_{14}{}^2\rho_{35} + (\rho_{23}+\rho_{12}\rho_{13})(\rho_{25}+\rho_{24}\rho_{45})\} + (2N+1)\rho_{12}\rho_{15}\rho_{24}\rho_{34}$$
$$+ (N+2)\rho_{14}(\rho_{15}\rho_{34}+\rho_{14}\rho_{23}\rho_{25}) + (N+3)\rho_{12}\rho_{14}\rho_{24}\rho_{35}$$
$$+ (N+4)\rho_{14}(\rho_{12}\rho_{25}\rho_{34}+\rho_{15}\rho_{23}\rho_{24})].$$

6 *variates*

$$(101)\quad \mu\begin{pmatrix} 1 & \cdot & \cdot & \cdot & \cdot & \cdot \\ & 1 & \cdot & \cdot & \cdot & \cdot \\ & & \cdot & 1 & \cdot & \cdot \\ & & & \cdot & \cdot & \cdot \\ & & & & \cdot & 1 \\ & & & & & \cdot \end{pmatrix} = 2\frac{N-1}{N^4}\sigma_1{}^2\sigma_2{}^2\sigma_3\sigma_4\sigma_5\sigma_6[(N-1)\{\rho_{12}{}^2(\rho_{35}\rho_{46}+\rho_{36}\rho_{45})$$
$$+ 2\rho_{13}\rho_{14}\rho_{25}\rho_{26} + 2\rho_{15}\rho_{16}\rho_{23}\rho_{24}\} + 2(\rho_{13}\rho_{24}+\rho_{14}\rho_{23})(\rho_{15}\rho_{26}+\rho_{16}\rho_{25})$$
$$+ 2\rho_{12}\rho_{13}(\rho_{25}\rho_{46}+\rho_{26}\rho_{45}) + 2\rho_{12}\rho_{14}(\rho_{25}\rho_{36}+\rho_{26}\rho_{35})$$
$$+ 2\rho_{12}\rho_{15}(\rho_{23}\rho_{46}+\rho_{24}\rho_{36}) + 2\rho_{12}\rho_{16}(\rho_{23}\rho_{45}+\rho_{24}\rho_{35})],$$

$$(102)\quad \mu\begin{pmatrix} 1 & 1 & \cdot & \cdot & \cdot & \cdot \\ & \cdot & \cdot & \cdot & \cdot & \cdot \\ & & \cdot & 1 & \cdot & \cdot \\ & & & \cdot & \cdot & \cdot \\ & & & & \cdot & 1 \\ & & & & & \cdot \end{pmatrix} = 2\frac{N-1}{N^4}\sigma_1{}^3\sigma_2\sigma_3\sigma_4\sigma_5\sigma_6[(N-1)\rho_{12}(\rho_{35}\rho_{46}+\rho_{36}\rho_{45}) + \rho_{13}(\rho_{25}\rho_{46}+\rho_{26}\rho_{45})$$
$$+ \rho_{14}(\rho_{25}\rho_{36}+\rho_{26}\rho_{35}) + \rho_{15}(\rho_{23}\rho_{46}+\rho_{24}\rho_{36}) + \rho_{16}(\rho_{23}\rho_{45}+\rho_{24}\rho_{35})$$
$$+ 2\rho_{12}\rho_{13}(\rho_{15}\rho_{46}+\rho_{16}\rho_{45}) + 2\rho_{12}\rho_{14}(\rho_{15}\rho_{36}+\rho_{16}\rho_{35})$$
$$+ (N+1)\{\rho_{13}\rho_{15}(\rho_{14}\rho_{26}+\rho_{16}\rho_{24}) + \rho_{14}\rho_{16}(\rho_{13}\rho_{25}+\rho_{15}\rho_{23})\}],$$

$$(103)\quad \mu\begin{pmatrix} 1 & \cdot & \cdot & \cdot & \cdot & \cdot \\ & \cdot & 1 & \cdot & \cdot & \cdot \\ & & \cdot & 1 & \cdot & \cdot \\ & & & \cdot & \cdot & \cdot \\ & & & & \cdot & 1 \\ & & & & & \cdot \end{pmatrix} = 2\frac{N-1}{N^4}\sigma_1{}^2\sigma_2\sigma_3{}^2\sigma_4\sigma_5\sigma_6[\rho_{12}(\rho_{15}\rho_{46}+\rho_{16}\rho_{45}) + \rho_{14}(\rho_{15}\rho_{26}+\rho_{16}\rho_{25})$$
$$+ 2\rho_{12}\rho_{14}\rho_{35}\rho_{36} + 2\rho_{13}\rho_{15}\rho_{24}\rho_{36} + 2\rho_{13}\rho_{16}\rho_{24}\rho_{35} + \rho_{13}{}^2(\rho_{25}\rho_{46}+\rho_{26}\rho_{45})$$
$$- N\{\rho_{12}\rho_{13}(\rho_{35}\rho_{46}+\rho_{36}\rho_{45}) + \rho_{13}\rho_{14}(\rho_{25}\rho_{36}+\rho_{26}\rho_{35})\} + \rho_{13}\rho_{15}(\rho_{23}\rho_{46}+\rho_{26}\rho_{34})$$
$$+ \rho_{13}\rho_{16}(\rho_{23}\rho_{45}+\rho_{25}\rho_{34}) + (N-1)\rho_{15}\rho_{16}(\rho_{24}+\rho_{23}\rho_{34})$$
$$+ (\rho_{12}\rho_{34}+\rho_{14}\rho_{23})(\rho_{15}\rho_{36}+\rho_{16}\rho_{35})],$$

$$(104)\quad \mu\begin{pmatrix} 1 & \cdot & \cdot & \cdot & \cdot & \cdot \\ & \cdot & 1 & \cdot & \cdot & \cdot \\ & & \cdot & \cdot & \cdot & \cdot \\ & & & \cdot & 1 & 1 \\ & & & & \cdot & \cdot \\ & & & & & \cdot \end{pmatrix} = 2\frac{N-1}{N^4}\sigma_1{}^2\sigma_2\sigma_3\sigma_4{}^2\sigma_5\sigma_6[\rho_{12}(\rho_{15}\rho_{36}+\rho_{16}\rho_{35}) + \rho_{13}(\rho_{15}\rho_{26}+\rho_{16}\rho_{25})$$
$$- (N-1)\rho_{12}\rho_{13}(\rho_{56}+\rho_{46}\rho_{45}) + N\rho_{14}\{\rho_{15}(\rho_{26}\rho_{34}+\rho_{24}\rho_{36})$$
$$+ \rho_{16}(\rho_{24}\rho_{35}+\rho_{25}\rho_{34})\} + \rho_{12}\rho_{14}(\rho_{35}\rho_{46}+\rho_{36}\rho_{45}) + \rho_{13}\rho_{14}(\rho_{26}\rho_{45}+\rho_{25}\rho_{46})$$
$$+ \rho_{14}{}^2(\rho_{25}\rho_{36}+\rho_{26}\rho_{35}) + 2\rho_{15}\rho_{16}\rho_{24}\rho_{34}$$
$$+ (\rho_{12}\rho_{34}+\rho_{13}\rho_{24})(\rho_{15}\rho_{46}+\rho_{16}\rho_{45}+2\rho_{14}\rho_{56})],$$

$$(105)\quad \mu\left(\begin{array}{r} \cdot\,2\,\cdot\,\cdot\,\cdot\,\cdot \\ \cdot\,\cdot\,\cdot\,\cdot\,\cdot \\ \cdot\,1\,\cdot\,\cdot \\ \cdot\,\cdot\,\cdot \\ \cdot\,1 \\ \cdot \end{array}\right) = \frac{N-1}{N^4}\sigma_1^2\sigma_2^2\sigma_3\sigma_4\sigma_5\sigma_6[(N-1)(1+\rho_{12}^2)(\rho_{35}\rho_{46}+\rho_{36}\rho_{45})$$
$$+ 2N(\rho_{13}\rho_{24}+\rho_{14}\rho_{23})(\rho_{15}\rho_{26}+\rho_{16}\rho_{25}) + 4(\rho_{13}\rho_{14}\rho_{25}\rho_{26}+\rho_{15}\rho_{16}\rho_{23}\rho_{24})$$
$$+ 2(\rho_{13}+\rho_{12}\rho_{23})(\rho_{15}\rho_{46}+\rho_{16}\rho_{45}) + 2(\rho_{14}+\rho_{12}\rho_{24})(\rho_{15}\rho_{36}+\rho_{16}\rho_{35})$$
$$+ 2(\rho_{23}+\rho_{12}\rho_{13})(\rho_{25}\rho_{46}+\rho_{26}\rho_{45}) + 2(\rho_{24}+\rho_{12}\rho_{14})(\rho_{25}\rho_{36}+\rho_{26}\rho_{35})],$$

$$(106)\quad \mu\left(\begin{array}{r} \cdot\,1\,1\,1\,\cdot\,\cdot \\ \cdot\,\cdot\,\cdot\,\cdot\,\cdot \\ \cdot\,\cdot\,\cdot\,\cdot \\ \cdot\,\cdot\,\cdot \\ \cdot\,1 \\ \cdot \end{array}\right) = \frac{N-1}{N^4}\sigma_1^2\sigma_2\sigma_3\sigma_4\sigma_5\sigma_6[2\rho_{12}(\rho_{35}\rho_{46}+\rho_{36}\rho_{45}) + 2\rho_{13}(\rho_{25}\rho_{46}+\rho_{26}\rho_{45})$$
$$+ 2\rho_{14}(\rho_{25}\rho_{36}+\rho_{26}\rho_{35}) + 4\rho_{15}\rho_{16}(\rho_{12}\rho_{34}+\rho_{13}\rho_{24}+\rho_{14}\rho_{23})$$
$$+ (N+1)\{(\rho_{23}+\rho_{12}\rho_{13})(\rho_{15}\rho_{46}+\rho_{16}\rho_{45}) + (\rho_{24}+\rho_{12}\rho_{14})(\rho_{15}\rho_{36}+\rho_{16}\rho_{35})$$
$$+ (\rho_{34}+\rho_{13}\rho_{14})(\rho_{15}\rho_{26}+\rho_{16}\rho_{25})\}],$$

$$(107)\quad \mu\left(\begin{array}{r} \cdot\,1\,1\,\cdot\,\cdot\,\cdot \\ \cdot\,\cdot\,1\,\cdot\,\cdot \\ \cdot\,\cdot\,\cdot\,\cdot \\ \cdot\,\cdot\,\cdot \\ \cdot\,1 \\ \cdot \end{array}\right) = \frac{N-1}{N^4}\sigma_1^2\sigma_2^2\sigma_3\sigma_4\sigma_5\sigma_6[(1+\rho_{12}^2)(\rho_{35}\rho_{46}+\rho_{36}\rho_{45})$$
$$+ (\rho_{13}+3\rho_{12}\rho_{23})(\rho_{15}\rho_{46}+\rho_{16}\rho_{45}) + (\rho_{24}+3\rho_{12}\rho_{14})(\rho_{25}\rho_{36}+\rho_{26}\rho_{35})$$
$$+ 2\rho_{34}(\rho_{15}\rho_{16}+\rho_{25}\rho_{26}) + 2(\rho_{13}\rho_{14}\rho_{25}\rho_{26}+\rho_{15}\rho_{16}\rho_{23}\rho_{24})$$
$$+ (\rho_{15}\rho_{26}+\rho_{16}\rho_{25})\{\rho_{13}\rho_{24}+(N+1)\rho_{12}\rho_{34}+(N+2)\rho_{14}\rho_{23}\}$$
$$+ N\{(\rho_{23}+\rho_{12}\rho_{13})(\rho_{25}\rho_{46}+\rho_{26}\rho_{45}) + (\rho_{14}+\rho_{12}\rho_{24})(\rho_{15}\rho_{36}+\rho_{16}\rho_{35})\}],$$

$$(108)\quad \mu\left(\begin{array}{r} \cdot\,\cdot\,1\,\cdot\,\cdot\,\cdot \\ \cdot\,1\,\cdot\,\cdot\,\cdot \\ \cdot\,\cdot\,\cdot\,\cdot \\ \cdot\,1\,1 \\ \cdot\,\cdot \\ \cdot \end{array}\right) = \frac{N-1}{N^4}\sigma_1\sigma_2\sigma_3^2\sigma_4^2\sigma_5\sigma_6[(N-1)(\rho_{12}+\rho_{13}\rho_{23})(\rho_{56}+\rho_{45}\rho_{46})$$
$$+ N\{(\rho_{14}\rho_{35}+\rho_{15}\rho_{34})(\rho_{24}\rho_{36}+\rho_{26}\rho_{34}) + (\rho_{14}\rho_{36}+\rho_{16}\rho_{34})(\rho_{24}\rho_{35}+\rho_{25}\rho_{34})\}$$
$$+ 2\rho_{12}\rho_{34}\rho_{36}\rho_{45} + \rho_{13}\rho_{24}(\rho_{35}\rho_{46}+\rho_{36}\rho_{45}) + \rho_{13}\rho_{26}(\rho_{35}+\rho_{34}\rho_{45})$$
$$+ 2\rho_{14}\rho_{34}(\rho_{25}\rho_{36}+\rho_{26}\rho_{35}) + 2\rho_{24}\rho_{34}(\rho_{15}\rho_{36}+\rho_{16}\rho_{35}) + 2\rho_{34}\rho_{56}(\rho_{12}\rho_{34}+\rho_{13}\rho_{24})$$
$$+ (\rho_{15}+\rho_{14}\rho_{45})(\rho_{26}+\rho_{23}\rho_{36}) + (\rho_{16}+\rho_{14}\rho_{46})(\rho_{25}+\rho_{23}\rho_{35})$$
$$+ (\rho_{24}+\rho_{23}\rho_{34})(2\rho_{14}\rho_{56}+\rho_{15}\rho_{46}+\rho_{16}\rho_{45}) + (\rho_{36}+\rho_{34}\rho_{46})(2\rho_{12}\rho_{35}+\rho_{13}\rho_{25})].$$

7 variates

$$(109)\quad \mu\left(\begin{array}{r} 1\,\cdot\,\cdot\,\cdot\,\cdot\,\cdot\,\cdot \\ \cdot\,1\,\cdot\,\cdot\,\cdot\,\cdot \\ \cdot\,\cdot\,\cdot\,\cdot\,\cdot \\ \cdot\,1\,\cdot\,\cdot \\ \cdot\,\cdot\,\cdot \\ \cdot\,1 \\ \cdot \end{array}\right) = 2\frac{N-1}{N^4}\sigma_1^2\sigma_2\sigma_3\sigma_4\sigma_5\sigma_6\sigma_7[\rho_{12}\rho_{14}(\rho_{36}\rho_{57}+\rho_{37}\rho_{56}) + \rho_{12}\rho_{15}(\rho_{36}\rho_{47}+\rho_{37}\rho_{46})$$
$$+ \rho_{12}\rho_{16}(\rho_{34}\rho_{57}+\rho_{35}\rho_{47}) + \rho_{12}\rho_{17}(\rho_{34}\rho_{56}+\rho_{35}\rho_{46}) + \rho_{13}\rho_{14}(\rho_{26}\rho_{57}+\rho_{27}\rho_{56})$$
$$+ \rho_{13}\rho_{15}(\rho_{26}\rho_{47}+\rho_{27}\rho_{46}) + \rho_{13}\rho_{16}(\rho_{24}\rho_{57}+\rho_{25}\rho_{47}) + \rho_{13}\rho_{17}(\rho_{24}\rho_{56}+\rho_{25}\rho_{46})$$
$$+ \rho_{14}\rho_{16}(\rho_{25}\rho_{37}+\rho_{27}\rho_{35}) + \rho_{14}\rho_{17}(\rho_{25}\rho_{36}+\rho_{26}\rho_{35}) + \rho_{15}\rho_{16}(\rho_{24}\rho_{37}+\rho_{27}\rho_{34})$$
$$+ \rho_{15}\rho_{17}(\rho_{24}\rho_{36}+\rho_{26}\rho_{34}) + (N-1)\{\rho_{12}\rho_{13}(\rho_{46}\rho_{57}+\rho_{47}\rho_{56})$$
$$+ \rho_{14}\rho_{15}(\rho_{26}\rho_{37}+\rho_{27}\rho_{36}) + \rho_{16}\rho_{17}(\rho_{24}\rho_{35}+\rho_{25}\rho_{34})\}],$$

$$(110)\quad \mu\left(\begin{array}{r} \cdot\,1\,1\,\cdot\,\cdot\,\cdot\,\cdot \\ \cdot\,\cdot\,\cdot\,\cdot\,\cdot\,\cdot \\ \cdot\,\cdot\,\cdot\,\cdot\,\cdot \\ \cdot\,1\,\cdot\,\cdot \\ \cdot\,\cdot\,\cdot \\ \cdot\,1 \\ \cdot \end{array}\right) = \frac{N-1}{N^4}\sigma_1^2\sigma_2\sigma_3\sigma_4\sigma_5\sigma_6\sigma_7[(\rho_{24}+\rho_{12}\rho_{14})(\rho_{36}\rho_{57}+\rho_{37}\rho_{56})$$
$$+ (\rho_{25}+\rho_{12}\rho_{15})(\rho_{36}\rho_{47}+\rho_{37}\rho_{46}) + (\rho_{26}+\rho_{12}\rho_{16})(\rho_{34}\rho_{57}+\rho_{35}\rho_{47})$$
$$+ (\rho_{27}+\rho_{12}\rho_{17})(\rho_{34}\rho_{56}+\rho_{35}\rho_{46}) + (\rho_{13}\rho_{24}+\rho_{14}\rho_{23})(\rho_{16}\rho_{57}+\rho_{17}\rho_{56})$$
$$+ (\rho_{13}\rho_{25}+\rho_{15}\rho_{23})(\rho_{16}\rho_{47}+\rho_{17}\rho_{46}) + (\rho_{13}\rho_{26}+\rho_{16}\rho_{23})(\rho_{14}\rho_{57}+\rho_{15}\rho_{47})$$
$$+ (\rho_{13}\rho_{27}+\rho_{17}\rho_{23})(\rho_{14}\rho_{56}+\rho_{15}\rho_{46}) + (\rho_{14}\rho_{26}+\rho_{16}\rho_{24})(\rho_{15}\rho_{37}+\rho_{17}\rho_{35})$$
$$+ (\rho_{14}\rho_{27}+\rho_{17}\rho_{24})(\rho_{15}\rho_{36}+\rho_{16}\rho_{35}) + (\rho_{15}\rho_{26}+\rho_{16}\rho_{25})(\rho_{14}\rho_{37}+\rho_{17}\rho_{34})$$
$$+ (\rho_{15}\rho_{27}+\rho_{17}\rho_{25})(\rho_{14}\rho_{36}+\rho_{16}\rho_{34}) + (N-1)\{(\rho_{23}+\rho_{12}\rho_{13})(\rho_{46}\rho_{57}+\rho_{47}\rho_{56})$$
$$+ (\rho_{14}\rho_{25}+\rho_{15}\rho_{24})(\rho_{16}\rho_{37}+\rho_{17}\rho_{36}) + (\rho_{14}\rho_{35}+\rho_{15}\rho_{34})(\rho_{16}\rho_{27}+\rho_{17}\rho_{26})\}].$$

8 *variates*

$$(111)\quad \mu\begin{pmatrix} \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ & & \cdot & 1 & \cdot & \cdot & \cdot & \cdot \\ & & & \cdot & \cdot & \cdot & \cdot & \cdot \\ & & & & \cdot & 1 & \cdot & \cdot \\ & & & & & \cdot & \cdot & \cdot \\ & & & & & & \cdot & 1 \\ & & & & & & & \cdot \end{pmatrix} = \frac{N-1}{N^4}\sigma_1\sigma_2\sigma_3\sigma_4\sigma_5\sigma_6\sigma_7\sigma_8\big[(\rho_{13}\rho_{25}+\rho_{15}\rho_{23})(\rho_{47}\rho_{68}+\rho_{48}\rho_{67})$$
$$+(\rho_{13}\rho_{26}+\rho_{16}\rho_{23})(\rho_{47}\rho_{58}+\rho_{48}\rho_{57})+(\rho_{13}\rho_{27}+\rho_{17}\rho_{23})(\rho_{45}\rho_{68}+\rho_{46}\rho_{58})$$
$$+(\rho_{13}\rho_{28}+\rho_{18}\rho_{23})(\rho_{45}\rho_{67}+\rho_{46}\rho_{57})+(\rho_{14}\rho_{25}+\rho_{15}\rho_{24})(\rho_{37}\rho_{68}+\rho_{38}\rho_{67})$$
$$+(\rho_{14}\rho_{26}+\rho_{16}\rho_{24})(\rho_{37}\rho_{58}+\rho_{38}\rho_{57})+(\rho_{14}\rho_{27}+\rho_{17}\rho_{24})(\rho_{35}\rho_{68}+\rho_{36}\rho_{58})$$
$$+(\rho_{14}\rho_{28}+\rho_{18}\rho_{24})(\rho_{35}\rho_{67}+\rho_{36}\rho_{57})+(\rho_{15}\rho_{27}+\rho_{17}\rho_{25})(\rho_{36}\rho_{48}+\rho_{38}\rho_{46})$$
$$+(\rho_{15}\rho_{28}+\rho_{18}\rho_{25})(\rho_{36}\rho_{47}+\rho_{37}\rho_{46})+(\rho_{16}\rho_{27}+\rho_{17}\rho_{26})(\rho_{35}\rho_{48}+\rho_{38}\rho_{45})$$
$$+(\rho_{16}\rho_{28}+\rho_{18}\rho_{26})(\rho_{35}\rho_{47}+\rho_{37}\rho_{45})+(N-1)\{(\rho_{13}\rho_{24}+\rho_{14}\rho_{23})(\rho_{57}\rho_{68}+\rho_{58}\rho_{67})$$
$$+(\rho_{15}\rho_{26}+\rho_{16}\rho_{25})(\rho_{37}\rho_{48}+\rho_{38}\rho_{47})+(\rho_{17}\rho_{28}+\rho_{18}\rho_{27})(\rho_{35}\rho_{46}+\rho_{36}\rho_{45})\}\big].$$