æ

Multivariate statistics with economic applications, Homework I.

1. Let $\mathbf{X}_1, \dots, \mathbf{X}_n$ be i.i.d. sample from a multivariate distribution, and assume that their common covariance matrix $\mathbf{C}$ exists. Prove that the empirical covariance matrix based on the sample is a strongly consistent estimator of $\mathbf{C}$!

2. Let $\mathbf{W} \sim \mathcal{W}_p(n, \mathbf{I}_p)$ be a $p \times p$ standard Wishart-matrix, $n > p$. Determine the distribution of $\mathrm{tr}\,(\mathbf{W})$!

3. Consider the random vector $(X_1, X_2, X_3)^T \sim \mathcal{N}_3(\underline{\mu}, \mathbf{C})$, where $\underline{\mu} = (\mu_1, \mu_2, \mu_3)^T$ and

$$\mathbf{C} = \begin{pmatrix} 1 & \rho & \rho^2 \\ \rho & 1 & 0 \\ \rho^2 & 0 & 1 \end{pmatrix}$$

   ($0 < \rho < 1$ is a given parameter). Determine the conditional distribution of $(X_1, X_2)^T$ conditioned on $X_3 = x_3$!

4. Consider the random vector $(X_1, X_2, X_3)^T \sim \mathcal{N}_3(\mathbf{0}, \mathbf{C})$, where

$$\mathbf{C} = \begin{pmatrix} 1 & \rho & \rho^2 \\ \rho & 1 & 0 \\ \rho^2 & 0 & 1 \end{pmatrix}$$

   ($0 < \rho < 1$ is a given parameter).

a. Find the multiple correlation between $(X_1, X_2)^T$ and X_3!

b. Find the partial correlation between X_1 and X_2, after eliminating the effect of X_3!

5. For the purchase- and selling prices of a certain currency in country A, based on 35 daily data, we obtained the following statistics: $\bar{\mathbf{X}} = (22.860, 24.397)^T$ and

$$\mathbf{S}_X = \begin{pmatrix} 17.178 & 19.710 \\ 19.710 & 23.710 \end{pmatrix}.$$

   For the same, in country B, based on 14 daily data, we obtained: $\bar{\mathbf{Y}} = (21.821, 22.843)^T$ and

$$\mathbf{S}_Y = \begin{pmatrix} 17.159 & 17.731 \\ 17.731 & 19.273 \end{pmatrix}.$$

   Assuming that the purchase- and selling prices follow 2-dimensional normal distribution in both countries with the same unknown covariance matrix, test the null-hypothesis that the expectation vector of the purchase- and selling prices is the same in the two countries!