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Multivariate statistics with economic applications, Homework II.

1. The performance of 4 firms is evaluated by 2-2-2-2 independent experts. The given scores (integers between 1 and 10) are the following:

Firm 1. : 6, 10

Firm 2. : 9, 5

Firm 3. : 9, 7

Firm 4. : 4, 6

Decide whether the 4 evaluations differ significantly with 95% confidence (0.05 significanvce). Please, write down the ANOVA decompositon as well.

2. Perform Principal Component Analysis on the covariance matrix

$$\mathbf{C} = \begin{pmatrix} 1 & 0.83 & 0.78 \\ 0.83 & 1 & 0.67 \\ 0.78 & 0.67 & 1 \end{pmatrix}.$$

Give the proportion of the total variance explained by the 3 principal components (each, separately)!

3. The cost of raw material and man-power to produce a certain product has empirical covariance matrix

$$\hat{\mathbf{C}}_{11} = \begin{pmatrix} 1.0 & 0.6328 \\ 0.6328 & 1.0 \end{pmatrix}.$$

The empirical covariance matrix of the retail and wholesale prices is

$$\hat{\mathbf{C}}_{22} = \begin{pmatrix} 1.0 & 0.4248 \\ 0.4248 & 1.0 \end{pmatrix},$$

whereas the empirical cross-covariance matrix of the costs and prices is

$$\hat{\mathbf{C}}_{12} = \begin{pmatrix} 0.2412 & 0.0586 \\ -0.0553 & 0.0655 \end{pmatrix}.$$

Decide by means of Cnonical Correlation analysis, how the costs and prices are correlated. Find the canonical correlations and investigate the following null-hypotheses (with significance 0.05) if $n = 148$.

a. The costs and prices are independent.

b. If a. rejected, then one canonical correlation together with the canonical vector pair suffices to explain the dependence between the costs and the prices.

4. The expectations (vectors) and common covariance matrix of three 2-dimensional normal populations are the following:

$$\underline{\mu}_1 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad \underline{\mu}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad \underline{\mu}_3 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} 5 & 2 \\ 2 & 1 \end{pmatrix}.$$

   Their a priori probabilities are 1/2, 1/3, 1/6. By using Discriminant Analysis, decide that to which population the observation $(0.5, 0.6)^T$ can be assigned.

5. The aggregated yearly GDP data (1970-79) in a country were the following:

$$66, 73, 80, 88, 96, 100, 108, 116, 123, 136.$$

   Estimate the first- and second-order autocovariances. Then fit an AR(2)-process to the data (estimate its coefficients)! Assume that the process is weakly stationary.