FUN Exercise sheet 10.

(112) (HW1) Let $T : H \to H$ be bounded self-adjoint, and let $m = \inf_{\|x\|=1}\langle Tx, x\rangle$. Prove that for any $\lambda \in \sigma(T)$ we have $\lambda \geq m$.

(113) Let A, B, T be bounded self-adjoint operators, such that $T \geq 0$ and T commutes with A and B. Show that $A \leq B$ implies $AT \leq BT$.

(114) Show that for any $T \in B(H, H)$ the operator T^*T is positive.

(115) (HW2) Let $T \geq 0$ be a positive operator. Prove that for all x, y we have $|\langle Tx, y\rangle| \leq \langle Tx, x\rangle^{1/2}\langle Ty, y\rangle^{1/2}$

(116) Prove that if $T \geq 0$ and $\langle Tx, x\rangle = 0$ then $Tx = 0$.

(117) Let S be unitary, P be a projection. Show that $S^{-1}PS$ is a projection.

(118) (HW3) Find the operator norm of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

(119) Let $S, T \geq 0$ be positive self-adjoint operators such that $ST = TS$. Prove that $TS \geq 0$. (You are allowed to use the existence of the square root operator.)

(120) Let S be unitary, P be a projection. Show that $S^{-1}PS$ is a projection.